Bank of Canada Banque du Canada

Working Paper 2004-42 / Document de travail 2004-42

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ISSN 1192-5434

Printed in Canada on recycled paper

105-00920/

Bank of Canada Working Paper 2004-42

November 2004

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The views expressed in this paper are those of the authors.
No responsibility for them should be attributed to the Bank of Canada, the European Central Bank, or the Eurosystem.

Contents

Acknowledgements . . . iv
Abstract/Résumé . . . v

1. Introduction . . . 1

2. Related Literature . . . 4
   2.1 Empirical work . . . 5
   2.2 Theoretical work . . . 6

3. The Model . . . 8
   3.1 Set-up . . . 8
   3.2 Stationary equilibria . . . 12

4. Characterizing Equilibrium Flows and Returns . . . 14
   4.1 The evolution of beliefs . . . 14
   4.2 Optimal portfolio choice . . . 17
   4.3 Equilibrium prices, predictability, and hedging . . . 19
   4.4 Equilibrium flows and returns . . . 22

5. Data . . . 25
   5.1 Dividends . . . 25
   5.2 Equity flows . . . 27

6. Quantitative Analysis . . . 31
   6.1 Calibration . . . 31
   6.2 Further predictions for flows and returns . . . 34
   6.3 Interpretation . . . 36

Bibliography . . . 40

Tables . . . 44

Figures . . . 50

Appendix A: Proof of Theorem 1 . . . 58

Appendix B: Detrending . . . 61

Appendix C: The Dividend Process . . . 63

Acknowledgements

We thank seminar participants at the University of Rochester WEGMANS conference, the 2003 SAET conference, the 2003 SED meeting, and the 2003 ES Winter Meetings. Albuquerque thanks the Lamfalussy Fellowship program, sponsored by the European Central Bank.

Abstract

The authors model trading by foreign and domestic investors in developed-country equity markets. The key assumptions are that (i) both the foreign and domestic investor populations contain investors of different sophistication, and (ii) investor sophistication matters for performance in both public equity and private off-market investments. A quantitative model with these assumptions delivers a unified explanation for three stylized facts about U.S. investors' international equity trades that have been documented in the literature: (i) trading by U.S. investors occurs in bursts of *simultaneous* buying and selling, (ii) Americans build and unwind foreign equity positions gradually, and (iii) U.S. investors increase their market share in a country when stock prices in that country have recently been rising.

JEL classification: F30, G12, G14, G15
Bank classification: Financial markets; International topics; Market structure and pricing

Résumé

Les auteurs modélisent l'activité boursière des investisseurs étrangers et nationaux dans les pays développés. Leur étude repose sur deux grandes hypothèses : i) le niveau de connaissance des marchés financiers varie parmi les investisseurs, tant étrangers que nationaux; ii) la familiarité des investisseurs avec le fonctionnement des marchés influe aussi bien sur le rendement des placements à la bourse que sur celui des placements privés hors marché. Le modèle quantitatif proposé permet d'expliquer de façon cohérente trois faits stylisés, mis en évidence dans la littérature, concernant l'activité boursière internationale des investisseurs américains. Premièrement, les transactions de ce groupe d'investisseurs prennent la forme de vagues *simultanées* d'achats et de ventes. Deuxièmement, ces investisseurs accroissent graduellement leurs participations à l'étranger et s'en défont tout aussi progressivement. Troisièmement, ils se portent acquéreurs de volumes accrus d'actions dans un pays donné quand les prix des valeurs mobilières y augmentent depuis peu.

Classification JEL : F30, G12, G14, G15
Classification de la Banque : Marchés financiers; Questions internationales; Structure de marché et fixation des prix

1. Introduction

Do differences in investor sophistication drive international equity flows? Existing literature has emphasized *cross-country* differences in information *about public equity*.[1] It is typically assumed that foreign investors have less information about domestic stocks than domestic investors. This view abstracts from two features of actual portfolio choice. First, country investor populations are heterogeneous. In modern, developed-country stock markets, where the best (and worst) foreign and local traders are likely to have very similar backgrounds and skills, within-country heterogeneity may be more important than cross-country heterogeneity. Second, many participants in public equity markets also invest in other risky assets. Differences in sophistication should be relevant not only for stock market performance, but also for profits made from private investment opportunities.

In this paper, we consider a model that accommodates both features and calibrate it to data on dividends, returns, and U.S. investors' trades in the G-7 countries. We show that this provides a unified explanation for a number of regularities that have been noted in the empirical literature, but that have not been captured in a structural model. In particular, data on U.S. investors' trades exhibit the following stylized facts:
(i) *Bursts of gross trading activity.* Gross aggregate purchases and sales by U.S. investors in a given country are positively contemporaneously correlated: trading by U.S. investors thus occurs in bursts of *simultaneous* buying and selling.
(ii) *Flow momentum (Persistence).* Americans build and unwind foreign positions gradually: a net inflow into a country observed today predicts a net inflow over at least the next 2 quarters.
(iii) *Return chasing.* Both current and lagged local stock returns are positively correlated with current net purchases by U.S. investors, normalized by market capitalization. The average U.S. investor thus "chases returns": when prices have been increasing, the average U.S. investor buys shares from the average local investor.

In our model, the stock market of a G-7 economy has domestic and U.S.-based participants. Unsophisticated investors, both domestic and American, trade stocks with (foreign or American) sophisticated investors. The latter are not only better informed about local stocks, but also have access to

[1] See section 2 for a review of the literature.

private "off-market" opportunities. Both dividends on stocks and payoffs on private opportunities depend on the local business cycle, but are not perfectly correlated. This set-up generates two motives for trade. First, the business cycle exposure of private investments creates a need for risk-sharing. Second, unsophisticated investors cannot distinguish between the price effects of market and off-market private information. This leads to *disagreement* about stock returns in equilibrium, even though unsophisticated investors rationally extract information from prices. Both motives for trade vary with the business cycle and thus entail persistent and cyclical equity flows.

To capture local business cycles, we estimate stochastic processes for local dividends. To infer the composition of investor populations, we use selected moments of U.S.-based investors' gross and net purchases, overall trading volume, and returns. We find that the populations of both local and U.S.-based participants are heterogeneous. In line with previous literature, the average U.S.-based participant has less local knowledge than the average local participant. The cross-country differences between average trades, however, is generally much smaller than the within-country difference between investor types.

To see how the model accounts for the stylized facts, consider a typical boom. As good news about the business cycle arrives, all investors update their assessment of future cash flows and stock prices begin to rise. At the same time, sophisticated investors increasingly locate profitable off-market opportunities. To exploit private opportunities without unduly increasing exposure to business cycle risk, they begin to sell stocks. With heterogeneous investor populations, this generates both volume and, in international data, a burst of gross trading activity. Moreover, since the average American is less sophisticated than the average local investor, the U.S. population is buying foreign stocks as prices are rising.

The above "risk-sharing trades" are slowed down by disagreement: unsophisticated investors who have less information about the state of the business cycle are initially less optimistic and will buy stocks only at a discount. A string of favourable returns can help convince them that a boom is in full swing. This leads, predictably, to a stronger inflow of unsophisticated, and hence also American, money chasing returns. In contrast, sophisticated investors withdraw increasing amounts from the stock market as the peak is approached. Only as the economy worsens and profitable private opportunities dry up do they return to the market. Again, the transition is slow as unsophisticated investors, who were overly optimistic at the peak, gradually

revise their opinion.

Except for the United Kingdom, the calibrated models do a good job in matching the autocorrelation function of U.S. investors' net purchases.[2] Indeed, the model predicts not only flow momentum (positive autocorrelation at short horizons of 1 to 3 quarters), but also *flow reversal*, that is, negative autocorrelation at longer horizons (5 to 7 quarters). This prediction derives from business cycle swings in trading; momentum and reversal are also features of the persistent component of dividends. In the data, there is strong evidence of flow reversal in Canada, France, and Germany, and somewhat weaker evidence of it in Japan and Italy. The models also do a good job for the cross-correlogram of flows and returns.

Return chasing is often cited as an example of "irrational" behaviour by uninformed foreign investors. This view is countered by Bohn and Tesar (1996), who construct estimates of expected local returns based on public information. They show that American investors tend to buy precisely when these expected returns are high. To further assess the performance of our model, we replicate the Bohn-Tesar exercise in our model economies. We consistently find positive correlation between expected returns conditional on public information and net purchases by U.S. investors. Our model may thus be viewed as providing further support to the "rational" view of return chasing.

Our model's ability to match the dynamics of equity flows relies on two features that distinguish it from most other asymmetric information set-ups. First, there are no noise traders. Many models use serially independent supply shocks ("noise trades") as a device to guarantee disagreement between traders in a rational-expectations equilibrium. However, noise trades are, by construction, reversed after one period. This implies that they induce *negative* serial correlation in net purchases, a fact not observed in the data. In our model, disagreement arises instead from an interplay of imperfect and asymmetric information. The true state of the business cycle is not perfectly observed by *any* investor. Since private opportunities are more profitable in booms, a high realized private return is a "good" private signal about the business cycle, and hence about future dividends. This induces positive correlation between *unexpected* private returns and stock returns. As a result, sophisticated investors' portfolio demands for stocks, and hence stock prices,

[2] We suspect that the failure of the model to fit U.K. data is due to the importance of London as an international financial centre. This is discussed later in more detail.

also depend on news about private opportunities that are orthogonal to the business cycle. Unsophisticated investors are unable to distinguish such news from business cycle shocks, which ensures disagreement. Since this mechanism relies on the imperfect observation of persistent factors, it is consistent with persistent trading activity.

Second, our model is based on fundamentals (the estimated dividend process) that exhibit momentum and reversal. It is often taken for granted that asymmetric information trivially generates serial correlation in flows regardless of what the fundamentals look like. This misleading intuition is based on finite-horizon models of dynamic trading. Given initial disagreement, such models generate a string of trades in the same direction as disagreement is gradually resolved through learning by uninformed investors. Importantly, this mechanism generates only *conditional* momentum in flows, *given* the initial disagreement. To calculate unconditional autocorrelations, one needs to take into account how the economy reached the initial state of disagreement. Our analysis clarifies that, if this occurs through a shock that quickly reverts to the mean, trades are also quickly reversed in equilibrium, which leads to *negative* unconditional autocorrelation! In our model, trades are instead driven by business cycle shocks that have a hump-shaped impulse response.[3]

This paper is organized as follows. Section 2 describes the related literature. Section 3 presents the model of equity trading. Section 4 discusses the properties of equilibrium stock flows and returns. Section 5 describes the data used in documenting the facts and in the calibration. The calibration and the quantitative results are reported in section 6. The appendixes solve the model, detrend the data, and estimate the dividend process.

2. Related Literature

Although there is a large empirical literature on the joint distribution of international equity flows and returns, there are relatively few theoretical

[3]The persistence of U.S. net purchases, which can be positive or negative, is harder to explain than the persistence of volume, which involves an absolute value. For example, a sequence of independently, identically distributed (i.i.d.) noise trader holdings would induce persistent volume, but successive net trades by investors would be negatively serially correlated. Similarly, Wang's (1994) model of trading volume, which is based on AR(1) fundamentals, generates persistent volume, but negative serial correlation in flows between investor types.

studies. We discuss both in turn.[4]

2.1 Empirical work

Two of the stylized facts we emphasize, flow momentum (persistence) and return chasing, are well known. Bohn and Tesar (1996) document persistence in the U.S. Treasury aggregate data that are also the basis for our calibration. Froot and Tjornhom (2002) examine persistence in international trades by individual mutual funds. Their analysis shows that the source of persistence in aggregate mutual fund investment is asynchronous trading across funds into individual countries. This result highlights the role of investor heterogeneity also emphasized in our model.

Bohn and Tesar (1996) first pointed out the return-chasing phenomenon, documenting positive contemporaneous correlation of flows and return at the quarterly frequency. Later work (Bohn and Tesar 1995; Brennan and Cao 1997; Choe, Kho, and Stulz 1999; Froot, O'Connell, and Seasholes 2001) shows that much of the contemporaneous correlation over longer periods is due to positive correlation of flows with lagged returns at higher frequencies. We thus consider the result that returns predict flows in our model to be more important than the mere contemporaneous correlation between flows and returns.

Positive correlation of gross purchases and sales is observed by Albuquerque, Bauer, and Schneider (2003), but has otherwise not received a great deal of attention. It is an important fact, since it essentially rules out a large class of models in international economics and finance in which representative agents in different countries trade country stock indexes or accumulate aggregate capital stocks.[5] The prevalence of bursts of gross trading activity suggests that this highly aggregated view is not an appropriate way to think about capital flows. In our model, gross trading activity is instead explained by heterogeneity of investor populations.

Additional evidence on investor heterogeneity comes from the literature on individual investor performance. There exists a large number of studies

[4] See Stulz (1999) for a comprehensive survey of both empirical and theoretical work.

[5] The only way for such models to be consistent with the flow data would be a strong time-aggregation effect. Albuquerque, Bauer, and Schneider (2003), however, document that positive correlation between gross purchases and sales also exists at the monthly frequency.

that use data on individual trades to examine whether local investors outperform foreigners or vice versa. This literature has not been conclusive, with strong results in both directions, depending on the time period and the data set used.[6] This is what one would expect if there is indeed investor heterogeneity. In addition, some studies provide direct evidence of heterogeneity. In Finnish data, Grinblatt and Keloharju (2000) find differences in trading behaviour between domestic household investors and domestic institutions. Choe, Kho, and Stulz (2001) analyze the trading behaviour of foreign investors (U.S. and others) and domestic institutions and individuals around days of significant abnormal returns and days of large buying or selling activity in Korea. They find that foreign investors trade at worse prices relative to domestic individuals, but not relative to domestic institutions.

2.2 Theoretical work

The structure of our model is similar to that in Wang's (1994) seminal paper on trading volume. In Wang's model, some agents who obtain private information also invest in a private asset. While the expected returns on the private asset are perfectly observed by informed investors and independent of dividends, a non-revealing rational-expectations equilibrium occurs if dividends are correlated with unexpected returns on the private asset. In contrast, our model relies on imperfect information by all investors and on a more general factor structure required to match the data. This gives rise to a different argument for non-revelation, as discussed above.

While there are a number of models of foreign equity holdings, in particular the home bias, the theoretical literature on *flows* is relatively recent.[7]

[6] For studies that suggest an advantage for domestic traders, see Frankel and Schmukler (1996) for Mexico, and Hau (2001) for Germany. Hamao and Mei (2001) find no significant evidence that foreigners are able to time the Japanese stock market.

In contrast, Seasholes (2000) finds that foreign investors in Taiwan systematically accumulate assets before positive earnings announcements, and systematically sell assets before negative earnings announcements. Bailey and Mao (2001) analyze periods of earnings and dividends announcements in Thailand and Singapore and find evidence consistent with foreigners having superior information compared with domestic residents. Froot and Ramadorai (2001) show that unexpected inflows into closed-end funds cause an increase in the prices of both the net asset value of the fund and that of the fund itself, indicating that foreign investors have significant private information. Karolyi (1999) shows that foreign investors have outperformed domestic investors in Japan.

[7] For a survey of the home-bias literature, see Lewis (1999).

Brennan and Cao (1997) first emphasized the contemporaneous correlation of flows and returns. In their model, foreign investors are less informed than domestic investors. This not only generates home bias, but it also implies that foreign investors react more to public information. If private information accumulates slowly, their model predicts positive contemporaneous correlation of foreigners' net purchases and returns, as in the data. The overreaction effect stressed by Brennan and Cao also exists in our model: unsophisticated investors mistake a temporary shock to dividends for a persistent shock and become net buyers. Since this type of shock is temporary, it is quickly reversed and contributes negatively to the autocorrelation of flows. For our calibrated models, variance decompositions show that this limits the contribution of temporary dividend shocks relative to the persistent business cycle shocks discussed above.

Brennan and Cao (1997) do not analyze the flow dynamics implied by their model. Similarly, Coval (1999), who studies a quantitative two-country model with asymmetrically informed investors, does not use his model to consider any of the stylized facts we examine. Hau and Rey (2002) develop a model of international equity flows in the presence of exchange rate risk and a price-elastic supply of foreign exchange, according to which a euro appreciation (say, relative to the U.S. dollar) decreases the excess supply of euros. Their model does well in explaining correlations between currency and equity returns. It fails to deliver positive contemporaneous correlation between foreign investors' net purchases and local returns, however, because foreign investors sell local equities when local equity returns are high but local currency returns are low.

Griffin, Nardari, and Stulz (2002) study a two-country model to explain the daily behavior of flows and returns in emerging markets. They generate return chasing by assuming that foreigners have "extrapolative expectations," which they argue could be caused by irrational or updating behaviour. They also emphasize wealth effects that make stock prices in a country depend on factors other than domestic fundamentals. In our model, unsophisticated investors optimally have extrapolative expectations, because stock prices depend on a factor that is uncorrelated with domestic dividends: the orthogonal component to the profitability of off-market opportunities.

3. The Model

In this section, we first describe a model of a small open economy in which sophisticated and unsophisticated investors trade stocks. We then derive expressions for various statistics of trading activity when investors belong to two heterogeneous populations identified by nationality. In particular, the population of U.S. investors contains both sophisticated and unsophisticated investors.

3.1 Set-up

3.1.1 Preferences

There is a continuum of infinitely lived investors. A fraction, ν_u, of investors is unsophisticated (indexed by u), while a fraction, $1-\nu_u$, is sophisticated (indexed by s). Investors have identical expected utility preferences that exhibit constant absolute risk aversion (CARA). At time t, an investor of type i ranks contingent consumption plans $\{c_t^i\}_{t=t}^{\infty}$ according to

$$-E\left[\sum_{t=t}^{\infty} \beta^{(t-t)} \exp^{-\gamma c_t^i} \,|\mathcal{I}_t^i\right], \tag{1}$$

where $\beta < 1$ is the discount factor, $\gamma > 0$ is the coefficient of absolute risk aversion, and $\mathcal{I}_t^i$ is the information set at time t, to be specified below.

3.1.2 Investment opportunities

There are three assets that are available to all investors. First, a risk-free bond pays a gross rate of return of R_f. Second, a risky "world asset" pays a per-dollar excess return of R_t^W in period t. Third, all investors participate in the domestic stock market. The single asset traded in this market is a claim to the dividend stream $\{D_t\}$. At date t, shares trade at a per-share ex-dividend price of P_t, and hence deliver a per-share excess return of $R_t^D = P_t - D_t - R_f P_{t-1}$. A single share is traded every period. A fourth asset is accessible to sophisticated investors alone; we refer to it as a private investment opportunity and denote its per-dollar excess return by R_t^B.

Dividends and asset returns are subject to both persistent and transitory shocks. Let F_t^D denote the persistent component of dividends. Returns on

private opportunities are predictable and the expected return is correlated with dividends: it is likely to depend on the local business cycle. Other fluctuations in the expected return on the private opportunity are summarized by a state variable, F_t^B, independent of F_t^D. Both state variables can depend on two lags of themselves. Letting $\mathbf{F}_t = \left(F_t^D, F_{t-1}^D, F_t^B, F_{t-1}^B\right)'$, the distribution of dividends and returns is summarized by[5]

$$D_t = \bar{D} + F_t^D + \varepsilon_t^D, \tag{2}$$

$$R_{t+1}^B = \bar{R}^B + \eta_D F_t^D + \eta_B F_t^B + \varepsilon_t^B, \tag{3}$$

$$R_t^W = \bar{R}^W + \varepsilon_t^W, \tag{4}$$

$$\mathbf{F}_t = \boldsymbol{\rho}\mathbf{F}_{t-1} + \boldsymbol{\varepsilon}_t^F. \tag{5}$$

Variables with bars denote unconditional means. All shocks are components of the vector process $\boldsymbol{\varepsilon}_t := \left(\boldsymbol{\varepsilon}_t^{Ft}, \varepsilon_t^D, \varepsilon_t^W, \varepsilon_t^B, \varepsilon_t^y\right)'$ that is serially uncorrelated and normally distributed with mean zero and covariance matrix $\sum_{\varepsilon\varepsilon}$. In addition, the matrices $\boldsymbol{\rho}$ and $E\left(\boldsymbol{\varepsilon}_t^F \boldsymbol{\varepsilon}_t^{Ft}\right)$ are block diagonal and $\boldsymbol{\varepsilon}_t^F$ is uncorrelated with all other shocks. The shock ε_t^y is described below.

3.1.3 Information

At date t, all investors know past and present stock prices and dividends, as well as returns on the world asset. The unsophisticated investors have no additional information; that is, $\mathcal{I}_t^u = \left\{P_{t-l}, D_{t-l}, R_{t-l}^W\right\}_{l=0}^{\infty}$. Sophisticated investors not only know $\mathcal{I}_t^u$, but they also observe (i) past and present returns on their private opportunities; (ii) the factor F_t^B; and (iii) a signal, $y_t^s = F_t^D + \varepsilon_t^y$, about the persistent component of dividends, where ε_t^y is uncorrelated with all other shocks. All sophisticated investors observe the same signals. They thus share the information set, $\mathcal{I}_t^s = \left\{P_{t-l}, D_{t-l}, R_{t-l}^W, R_{t-l}^B, F_{t-l}^B, y_{t-l}^s\right\}_{l=0}^{\infty}$.

3.1.4 Portfolio choice

The budget constraint of investor i at date t is

$$w_{t+1}^i = R_f\left(w_t^i - c_t^i\right) + \boldsymbol{\theta}_t^{i\prime}\mathbf{R}_{t+1}^i, \tag{6}$$

where w_t^i is beginning-of-period wealth and the vectors $\boldsymbol{\theta}_t^i$ and $\mathbf{R}_t^i$ denote holdings and returns of assets that are available to investor i. In particular,

[5] We use bold print for vectors and matrices.

for sophisticated investors, $\boldsymbol{\theta}_t^s = \left(\theta_t^{Ds}, \theta_t^{Ws}, \theta_t^{Bs}\right)'$ and $\mathbf{R}_t^s = \left(R_t^D, R_t^W, R_t^B\right)'$; for unsophisticated investors, $\boldsymbol{\theta}_t^u = \left(\theta_t^{Du}, \theta_t^{Wu}\right)'$ and $\mathbf{R}_t^u = \left(R_t^D, R_t^W\right)'$. Investor i chooses contingent plans for consumption, $\{c_t^i\}_{t=t}^{\infty}$, and asset holdings, $\{\boldsymbol{\theta}_t^i\}_{t=t}^{\infty}$, to maximize expected utility (1), conditional on the information set $\mathcal{I}_t^i$ and the budget constraints (6).

3.1.5 Equilibrium

A rational-expectations equilibrium is a collection of stochastic processes $\{c_t^u, c_t^s, \boldsymbol{\theta}_t^u, \boldsymbol{\theta}_t^s, P_t\}$ for consumption, asset holdings, and the domestic stock price such that (i) both types of agents choose optimal portfolios and consumption, given prices, and (ii) the domestic stock market clears:

$$\nu_u \theta_t^{Du} + (1 - \nu_u)\,\theta_t^{Ds} = 1. \tag{7}$$

A key feature of this equilibrium is that agents look at current and past prices to update their beliefs about variables they do not observe. In particular, unsophisticated investors will try to learn from prices about the return on private opportunities and the signal y_t^s, which sophisticated investors get about the persistent component of dividends.

3.1.6 International equity flows

To apply the model to data on U.S. investors' trades in international markets, we assume that there are two investor nationalities. One consists of all investors that have accounts based in the United States. We refer to the other group as "local" investors. It is natural to permit both the United States and the local population to contain both sophisticated and unsophisticated types. Let ν_{US} denote the measure of U.S. investors and let ν_{uUS} denote the fraction of unsophisticated U.S. investors relative to all U.S. investors. Aggregate U.S. holdings of the local asset are given by

$$\theta_t^{DUS} = \nu_{US}\left[\nu_{uUS}\theta_t^{Du} + (1 - \nu_{uUS})\,\theta_t^{Ds}\right].$$

In our model, trade is due only to the heterogeneity of sophisticated and unsophisticated investors. The market-clearing condition (7) thus implies that we can write all relevant statistics in terms of the holdings or trades of one type. We choose to express everything in terms of unsophisticated investors' holdings. For example, U.S. holdings of local equities can be written

as

$$\theta_t^{DUS} = \nu_{US}\left[\frac{1-\nu_{dUS}}{1-\nu_u} - \frac{\nu_{dUS}-\nu_u}{1-\nu_u}\theta_t^{Du}\right]. \tag{8}$$

3.1.7 Remarks

Our model differs from standard small open-economy models in that the expected return on the domestic stock market is endogenous, while the riskless rate, the world asset return, and the return on the off-market asset are taken as exogenous. In other words, we do not assume that there is one (exogenous) pricing kernel that can be used to price all assets. The simplest way to interpret our set-up is that there is market segmentation. The domestic market is used by domestic investors as well as by a subset of U.S. investors who are themselves small relative to the U.S. market. The world asset (say, the U.S. stock market index) is priced by the majority of U.S. investors who do not participate in the country under consideration.

Our approach thus *assumes* that equity home bias exists, and that it exists because of limited U.S. participation in foreign markets. Our goal is not to explain the world distribution of holdings of all assets, but trades in the stock market under consideration, conditional on home bias. We thus model participants in only that market. We also make the simplifying assumption that the world return is unpredictable and that unexpected world returns are uncorrelated with any other shock. That is, ε_t^W is independent of all other shocks in $\boldsymbol{\varepsilon}_t$. This assumption is counterfactual for industrial countries, and it could be relaxed to accommodate a common factor in returns and fundamentals.[9] However, it is not clear how important this extension would be for the properties of *flows* we are interested in. The mechanisms stressed below would still be present in the richer model. Moreover, Bohn and Tesar (1996) document that there is only a weak relationship between U.S. investors' international equity flows and U.S. equity returns.

We refer broadly to the fourth asset as "private investment opportunities," which means investment opportunities that: (i) become available to a subset of market participants that is also well-informed about the market itself, and (ii) are too costly to observe and access for all other market participants. Concrete examples of such opportunities abound in private equity, real estate, foreign exchange, or derivatives markets. Importantly, our model

[9] See Dumas, Harvey, and Ruiz (2002) for a model of world stock returns and output that emphasizes cross-country correlation.

does not require that the type of opportunity always be the same. All that matters is that, from time to time, the well-informed part of the population discover some new way to make money that is not known to everybody.

Lack of knowledge by unsophisticated investors can have different meanings. One possibility is simply that the private opportunity is secret. More generally, one can think of unsophisticated investors as people who concentrate on only a subset of the available public information. Even though, in principle, there may be data on the latest "hot" opportunity that sophisticated investors exploit, unsophisticated investors, who are not sure where to look, prefer to focus on stock market information that they know how to process. In our model, they process this information optimally: they know the stochastic processes for prices and update their beliefs by Bayes' rule. The ability of sophisticated investors to recognize investment opportunities that are not readily (or costlessly) available to unsophisticated investors is also included in Merton (1987) and Shapiro (2002).

We have assumed that sophisticated investors have better information about the persistent component of dividends. Sophisticated investors are thus agents who are better at analyzing medium-term prospects. This is an important part of our set-up. An alternative assumption would be to let the signal depend simply on future dividends. In that case, information would concern at least in part the short-term noise in dividends. More trades based on private information would follow news about noise, inducing more negative serial correlation in flows. In our calibration below, we decompose our estimated dividend process into a persistent and a transitory component. The precision of the signal y_t^s then regulates the knowledge of sophisticated investors relative to the econometrician.

3.2 Stationary equilibria

Let $\hat{\mathbf{F}}_t^i = E\left[\mathbf{F}_t|\mathcal{I}_t^i\right]$ denote investor i's conditional expectation of the vector $\mathbf{F}_t$ that drives persistent movements in fundamentals. Since $\mathcal{I}_t^u \subset \mathcal{I}_t^s$, the law of iterated expectations implies $\hat{\mathbf{F}}_t^u = E\left[\hat{\mathbf{F}}_t^s|\mathcal{I}_t^u\right]$. In other words, $\hat{\mathbf{F}}_t^u$ is the unsophisticated investors' expectation of what sophisticated investors expect $\mathbf{F}_t$ to be. We focus on equilibria in which the price can be written as a linear function of these expectations:

$$P_t = \bar{\pi} + \boldsymbol{\pi}_s' \hat{\mathbf{F}}_t^s + \boldsymbol{\pi}_u' \hat{\mathbf{F}}_t^u. \tag{9}$$

for some constants $\bar{\pi}$, $\boldsymbol{\pi}_s$, and $\boldsymbol{\pi}_u$.

Theorem 1 *There exists a rational expectations equilibrium such that the price satisfies (9). Equilibrium prices and asset holdings are stationary. Investor i's equilibrium stock holdings take the form*

$$\theta_t^{D_i} = \bar{\theta}^{D_i} + \boldsymbol{\Theta}^{D_i} \hat{\mathbf{F}}_t^u. \tag{10}$$

The equilibrium has two important properties. First, equilibrium prices do not reflect the true values of the persistent components of dividends or private returns, but only investors' perceptions of them. This differs from Wang (1994), where some investors have full information and are likely to generate additional trading volume as the asymmetry of information across agents is decreased. Second, holdings of both sophisticated and unsophisticated investors depend only on unsophisticated investors' estimates of the persistent factors, $\hat{\mathbf{F}}_t^u$. Trading of sophisticated investors thus differs from trading of unsophisticated investors because of the weights placed on each of these factors. We return to this later.

Appendix A provides a complete proof of Theorem 1. The main argument can be summarized as follows. Consider first the agents' payoff-relevant information. Suppose the information sets $\mathcal{I}_t^i$ contain only normal random variables. This implies normality of the conditional expectations, $\hat{\mathbf{F}}_t^i$, and, if the price satisfies (9), normality of all per-share returns. It follows that $\boldsymbol{\phi}_t^s = \left(\hat{\mathbf{F}}_t^{s\prime}, \hat{\mathbf{F}}_t^{u\prime}\right)'$ is a sufficient statistic for forecasting all future returns, given the information set $\mathcal{I}_t^s$.

Similarly, $\boldsymbol{\phi}_t^u = \hat{\mathbf{F}}_t^u$ is a sufficient statistic for forecasting returns given the information set $\mathcal{I}_t^u$. This includes one-step-ahead returns, since the current price can be written as a function of $\hat{\mathbf{F}}_t^u$. Indeed, unsophisticated investors know $\hat{\mathbf{F}}_t^u$, so that observing the price is the same as observing the signal

$$y_t^u = P_t - \bar{\pi} - \boldsymbol{\pi}_u' \hat{\mathbf{F}}_t^u = \boldsymbol{\pi}_s' \hat{\mathbf{F}}_t^s.$$

But then, $\boldsymbol{\pi}_s' \hat{\mathbf{F}}_t^u = E\left[\boldsymbol{\pi}_s' \hat{\mathbf{F}}_t^s | \mathcal{I}_t^u\right] = \boldsymbol{\pi}_s' \hat{\mathbf{F}}_t^s$, and we can write the price as $P_t = \bar{\pi} + (\boldsymbol{\pi}_s' + \boldsymbol{\pi}_u') \hat{\mathbf{F}}_t^u$. It follows that the state vector $\boldsymbol{\phi}_t^i$ captures the payoff-relevant information of investor i's problem regarding consumption, savings, and portfolio choice.

An important feature of exponential utility is that optimal portfolios are independent of wealth and linear in the agents' state vector: $\boldsymbol{\theta}_t^i = \bar{\boldsymbol{\theta}}^i -$

$\Theta^{\phi}\phi_t$. The coefficients $\bar{\theta}^{\phi}$ and Θ^{ϕ} will typically depend on the distribution of the exogenous variables, as well as the price coefficients $\bar{\pi}$, π_y, and π_s. The equilibrium condition requires that the price coefficients satisfy

$$\begin{aligned} \nu_y \bar{\theta}^{D_0} + (1-\nu_y)\bar{\theta}^{D_S} &= 1, \\ \nu_y \Theta^{D_0} + (1-\nu_y)\Theta^{D_S}_{I^u} &= 0, \end{aligned}$$

where $\Theta^{D_S}_{I^u}$ is the subvector of Θ^{D_S} that corresponds to $\hat{\mathbf{F}}^u_t$, with the remaining elements of the vector Θ^{D_S} set to zero. Finding an equilibrium thus boils down to solving a non-linear system of equations in the price coefficients.

4. Characterizing Equilibrium Flows and Returns

In this section, we discuss analytically some properties of equilibria. We first discuss how beliefs evolve and why disagreement can persist in equilibrium. We then establish properties of stock pricing. Finally, we calculate statistics that we use to calibrate the model and evaluate its account of the stylized facts.

4.1 The evolution of beliefs

Investors in our model economy continually learn about the state of the business cycle and the availability of private opportunities from observing prices, dividends, and private signals. Since all state variables are normal and homoscedastic, the evolution of investors' beliefs can be described by tracking conditional expectations, using the Kalman filter. The resulting equations clarify why disagreement can arise in equilibrium and how different agents over- or underestimate shocks.

4.1.1 Filtering

Sophisticated investors learn about the state of the business cycle by observing dividends, returns on their private opportunities, and their private signal. They do not learn from the price, since they already know $\hat{\mathbf{F}}^s_t$ and hence $\hat{\mathbf{F}}^y_t$. We collect their "relevant" observables in a vector, $\mathbf{o}^s_t =$

$(D_t - \bar{D},\ y_t^s,\ R_t^B - \bar{R}^B - \eta_B F_t^B)$, that can be represented as[10]

$$\mathbf{o}_t^s = \mathbf{M}^{osf}\mathbf{F}_{t-1} + \mathbf{M}^{ose}\varepsilon_t. \tag{11}$$

Equations (5) and (11) form a state-space system. Sophisticated investors' conditional expectation of the state vector, $\hat{\mathbf{F}}_t^s$, then takes the form

$$\begin{aligned}\hat{\mathbf{F}}_t^s &= \boldsymbol{\varrho}\hat{\mathbf{F}}_{t-1}^s + \mathbf{K}^s\left(\mathbf{o}_t^s - \mathbf{M}^{osf}\hat{\mathbf{F}}_{t-1}^s\right)\\ &= \boldsymbol{\varrho}\hat{\mathbf{F}}_{t-1}^s + \hat{\boldsymbol{\varepsilon}}_t^s,\end{aligned} \tag{12}$$

where $\mathbf{K}^s$ is a steady-state Kalman gain matrix.[11]

Unsophisticated investors obtain valuable information from dividends as well as from the signal, y_t^u, contained in prices: i.e., $\mathbf{o}_t^u = (D_t - \bar{D},\ y_t^u)$. These variables can be represented using $\hat{\mathbf{F}}_t^s$:[12]

$$\mathbf{o}_t^u = \mathbf{M}^{ouf}\hat{\mathbf{F}}_{t-1}^s + \mathbf{M}^{ose}\hat{\boldsymbol{\varepsilon}}_t^s. \tag{13}$$

Equations (12) and (13) form the state-space system of unsophisticated investors. Their conditional expectation, and hence their state variable, $\boldsymbol{\phi}_t^u$, can be written as

$$\begin{aligned}\hat{\mathbf{F}}_t^u &= \boldsymbol{\varrho}\hat{\mathbf{F}}_{t-1}^u + \mathbf{K}^u\left(\mathbf{o}_t^u - \mathbf{M}_{t-1}^{ouf}\hat{\mathbf{F}}_{t-1}^u\right)\\ &= \left(\boldsymbol{\varrho} - \mathbf{K}^u\mathbf{M}^{ouf}\right)\hat{\mathbf{F}}_{t-1}^u + \mathbf{K}^u\mathbf{M}^{ouf}\hat{\mathbf{F}}_{t-1}^s + \mathbf{K}^u\mathbf{M}^{ose}\hat{\boldsymbol{\varepsilon}}_t^s.\end{aligned} \tag{14}$$

The law of motion of sophisticated investors' state variable, $\boldsymbol{\phi}_t^s$, is summarized by (12) and (14).

4.1.2 Non-revealing prices

Since the stock price acts as a signal, the information structure in the model is endogenous. We say that investors *agree about the stock market* if their conditional distributions of future stock payoffs are the same. This

[10] Because the world return, R_t^W, is uncorrelated with everything else, it does not add any relevant information.

[11] Importantly, the matrix $\mathbf{M}^{ose}$ allows errors in the observation equation to be correlated with errors in the state equation.

[12] The matrices are $M^{ouf} = \begin{pmatrix} \boldsymbol{\pi}_s'\boldsymbol{\varrho} \\ \mathbf{M}_1^{osf} \end{pmatrix}$ and $M^{ose} = \begin{pmatrix} \boldsymbol{\pi}_s' \\ \mathbf{e}_1 \end{pmatrix}$, where $\mathbf{e}_1$ is the first unit vector.

is certainly true in the symmetric information benchmark, where investors are assumed to agree on all state variables: $F_t^u = F_t^s$. Agreement about the stock market, however, could also arise endogenously in our asymmetric information set-up if prices were to reveal all relevant information about stocks.[13]

Agreement about the stock market cannot occur in the linear equilibrium of Theorem 1. In our set-up, equilibria are "non-revealing," because (i) the business cycle component (F_t^D) is not perfectly observed by the sophisticated investor, and (ii) the expected private return depends on the business cycle. Private returns (R_t^B) are a signal of the state of the business cycle, and surprise moves in R_t^B change the conditional expectations $\hat{F}_t^{D,s}$ and $\hat{F}_{t-1}^{D,s}$. Because prices depend (at least) on these variables, sophisticated investors must perceive *unexpected* returns on stocks and private opportunities as correlated.[14] This implies that the price cannot be independent of *expected* private returns, and hence $\hat{F}_t^{B,s}$.

With a price that depends on both $\hat{F}^{D,s}$ and $\hat{F}^{B,s}$, unsophisticated investors cannot distinguish signals about the business cycle from signals relevant to private returns only. Suppose that, initially, agents were in agreement: $\hat{F}_{t-1}^u = \hat{F}_{t-1}^s$. By (12), sophisticated investors would then update according to the four-dimensional innovation vector, $\tilde{z}_t^s$. Unsophisticated investors would observe only the pair $\left(\pi_s' \tilde{z}_t, \tilde{z}_{t-1}^s\right)$. For example, high prices could signal either good news about dividends or bad news about private return opportunities.

4.1.3 Disagreement about the state of the business cycle

Investors' opinions about the state of the business cycle, $\left(\hat{F}_t^{D,i}, \Delta\hat{F}_t^{D,i}\right)$, $i = u, s$, are key determinants of equilibrium flows and returns. The Kalman filter equations show how these conditional expectations react to shocks. We say that an investor *overreacts* (underreacts) to a shock if $\hat{F}^{D,i}$ moves more (less) than the actual state variable, F^D. As a general rule, inference

[13] Agreement about the stock market is thus weaker than symmetric information. It already occurs if $\hat{F}_t^{D,u} = F_t^{D,s}$ and the stock price is independent of $\hat{F}^{B,s}$ and $\hat{F}^{B,u}$, even though unsophisticated investors do not know about private returns (which are not relevant to them).

[14] We do not require correlation between dividend shocks and unexpected returns on private opportunities under the true distribution. This is in contrast to Wang (1994), where this correlation is key to obtaining non-revelation. His model does not have the features (i) and (ii) noted for our model.

about slow-moving state variables from data contaminated by temporary noise induces overreaction to temporary shocks, but underreaction to persistent shocks to the state variable. In our model, both types of investors have imperfect information about F^D and will thus overreact to ε_t^D and ε_t^B, but underreact to ε_t^{FD}.

With asymmetric information, shocks also induce disagreement. For example, consider a positive shock to the persistent component of dividends (ε_t^{FD}). It is reflected in the dividend, observed by both investors. In addition, sophisticated investors obtain information about it from their private signal. In contrast, unsophisticated investors see only the indirect signal contained in the price. In a non-revealing equilibrium, this is contaminated by other shocks. Sophisticated investors underreact less: they underestimate F_t^D by less than sophisticated investors. As a result, sophisticated investors become more optimistic. The response to a positive *temporary* shock to dividends produces the opposite result: both investors see the dividend increase, but sophisticated investors do *not* see an unusual movement in their private signal. This causes them to assign lower probability to the fact that F_t^D has moved. It follows that unsophisticated investors become more optimistic.

A positive persistent shock to private returns is fully observed by sophisticated investors. Unsophisticated investors, on the other hand, see only a noisy signal of it through a lower price level. Because the lower stock price could also have been caused by a negative business cycle shock, unsophisticated investors end up underestimating the business cycle, increasing disagreement. Finally, a temporary shock to private returns will generate a noisy signal of the business cycle to sophisticated investors—as they observe the private return—and to unsophisticated investors—as they observe the stock price move. Such a shock causes sophisticated investors to underestimate the business cycle by more if unexpected shocks to dividends and private returns are positively correlated.

4.2 Optimal portfolio choice

In Appendix A, we solve investor i's consumption and portfolio choice problem, given the law of motion for ϕ_t^i. The value function is

$$V\left(w_t^i; \boldsymbol{\phi}_t^i\right) = -\exp\left[-\kappa^i - \tilde{\gamma} w_t^i - \mathbf{u}_i' \boldsymbol{\phi}_t^i - \frac{1}{2} \boldsymbol{\phi}_t^{i\prime} \mathbf{U}_i \boldsymbol{\phi}_t^i\right], \tag{15}$$

where $\tilde{\gamma}_i = \gamma \frac{R_f - 1}{R_f}$ and $\mathbf{U}_i$ is positive definite. Risk-averse investors care not only about fluctuations in wealth, but also about changes in beliefs, captured by the state vector $\boldsymbol{\phi}_t^i$. The quadratic term reflects investors' taste for "unusual" investment opportunities.[17]

With this value function, portfolio demand is linear in investors' state variables. To gain intuition about equilibrium holdings and trades, let $X_{t+1} = P_{t+1} + D_{t+1}$ denote the *payoff* on stocks, and define the conditional moments $\sigma_u^2 = var_t^u(X_{t+1})$, $\sigma_s^2 = var_t^s(X_{t+1})$, $\sigma_b^2 = var_t^s\left(R_{t+1}^b\right)$, and $\rho_s = corr_t^s\left(X_{t+1}, R_{t+1}^B\right)$, where *adjusted* conditional distributions are used for both agents.[18] We then have

$$\theta_t^{Du} = \frac{1}{\gamma \sigma_u^2} \left(\underbrace{E_t^u X_{t+1} - RP_t}_{\text{myopic demand}} + \underbrace{\bar{h}^u - \mathbf{H}^u \boldsymbol{\phi}_t^u}_{\text{hedging demand}} \right). \tag{16}$$

$$\theta_t^{Ds} = \frac{1}{\gamma \sigma_s^2 (1 - \rho_s^2)} \cdot \left(\underbrace{E_t^s X_{t+1} - RP_t - \rho_s \frac{\sigma_s}{\sigma_b} E_t^s R_{t+1}^b}_{\text{myopic demand}} + \underbrace{\bar{h}^s - \mathbf{H}^s \boldsymbol{\phi}_t^s}_{\text{hedging demand}} \right). \tag{17}$$

For both types, the first term captures responses to changes in one-period-ahead expected excess returns. It is relevant even if investors are myopic. Unsophisticated investors' myopic demand is simply proportional to expected per-share stock returns. In contrast, as long as stock and private returns are correlated, sophisticated investors' myopic demand also depends on expected private returns. In our numerical examples, stocks and private opportunities are substitutes ($\rho_s > 0$), since they move together with the business cycle. This tends to lower sophisticated investors' demand for stocks.

The "intertemporal hedging demand" of the investors is due to their concern with movements in the state variables, $\boldsymbol{\phi}_t^i$. Investor i effectively behaves

[17] Some intuition can be obtained by considering the case of one state variable. In this case, $\mathbf{U}_i$ is a positive number and continuation utility is higher the further $\boldsymbol{\phi}_t^i$ is from its mean of zero. Since $\boldsymbol{\phi}_t^i$ is payoff-relevant, it drives expected returns at some time in the future. An unusual value signals that above-average expected returns will be available, by going either long or short.

[18] In the current discrete-time setting, the covariance matrix of returns has to be adjusted to account for agents' taste for future unusual state variables as measured through the vector $\boldsymbol{\phi}_t^i$. This is described in detail in Appendix A.

as if they were holding a portfolio of non-tradable assets with return vector $\boldsymbol{\phi}^i_{t+1}$. Under this interpretation, the time-varying vector of shares held in each state variable is $\mathbf{u}_i - \mathbf{U}'_i E^i\left[\boldsymbol{\phi}^i_{t+1}|\boldsymbol{\phi}^i_t\right]$. Since investors fear states of poor investment opportunities, they favour assets that pay off in precisely these states: the average hedging demand, $\bar{h}^i$, is particularly high for such assets. Moreover, since investors desire unusual opportunities, their exposure to a state variable increases if that state variable is expected to take on an unusual value.[17] This gives rises to the time-varying hedging demand, $\mathbf{H}^i\boldsymbol{\phi}^i_t$.

4.3 Equilibrium prices, predictability, and hedging

In our numerical results below, (i) the local stock price depends strongly and positively on the level and change in the local business cycle, (ii) the local stock price depends weakly and negatively on the level and change in the off-market factor, and (iii) consideration of intertemporal hedging is crucial for understanding the behaviour of sophisticated investors, whereas it is largely irrelevant for unsophisticated investors. These properties of the model are closely connected. To see this, it is helpful to first write the equilibrium price as a weighted average of two hypothetical prices, P^u_t and P^s_t, that would arise in economies inhabited by only one type of agent.

4.3.1 Decomposition of a stock price

Using (16), (17), and the market-clearing condition for local stocks, we obtain[18]

$$\begin{aligned} P_t &= \tilde{\nu}_u P^u_t + (1-\tilde{\nu}_u) P^s_t \\ P^u_t &= \beta E^u_t X_{t+1} - \beta\left(\gamma\sigma^2_u - \left(\bar{h}^u - \mathbf{H}^u\boldsymbol{\phi}^u_t\right)\right) \\ P^s_t &= \beta E^s_t X_{t+1} - \beta\left(\gamma\sigma^2_s\left(1-\rho^2_s\right) - \left(\bar{h}^s - \mathbf{H}^s\boldsymbol{\phi}^s_t\right)\right) \\ &\quad - \beta\rho_s\frac{\sigma_s}{\sigma_b}E^s_t R^b_{t+1}. \end{aligned} \tag{18}$$

The price P^u_t behaves like the price in a representative-agent model with no private opportunities: it equals the present discounted payoff minus a

[17] More generally, with a vector of state variables, exposure to, say, the first element increases if "complementary" elements are expected to be high. Complementary elements are those for which the product with the first element yields high utility.

[18] Of course, the payoffs and the distribution of the state variables would also be different in the hypothetical representative-agent economies. The point is that the *structure* of the price equations is the same.

risk premium that consists of a constant "myopic" premium, $\bar{\gamma}\gamma\sigma_u^2$, less the intertemporal hedging demand. This suggests that *the presence of unsophisticated investors tends to reduce time variation in risk premiums*. Indeed, since unsophisticated investors have no access to private opportunities, their hedging demand can come only from the predictability of excess local *stock* returns. If expected excess returns are close to constant, the same is true for the hedging demand. By (18), price changes will then mostly reflect changes in the expected present value, and expected excess returns must indeed be close to constant. This logic implies low predictability for an actual representative-agent economy with unsophisticated investors. The result carries over to our model if the number of unsophisticated investors is large enough.

In contrast, the price P_t^s behaves like the price in an economy where all stockholders are entrepreneurs who run a private business in addition to investing in the stock market. The risk premium contains an additional "myopic" component that depends on the time-varying expected private return. Since ρ_s is positive in the equilibria we consider, this premium is also positive.[19] Equation (18) clarifies how *predictability in private returns can spill over to the stock market to produce time variation in expected stock returns*. With positive perceived correlation between private and public equity returns, sophisticated investors who face temporarily high expected private returns will want less exposure to business cycle risk common to both assets, and hence demand higher risk premiums on stocks.[20] In addition, since entrepreneurs optimize dynamically, their hedging demand depends on the correlation of stock returns with the future investment opportunities. This can further contribute to time variation in risk premiums.

The weight, $\bar{\nu}_u$, on the "unsophisticated price," P_t^u, depends on the unsophisticated investors' overall ability to "move the market." It therefore depends not only (positively) on the number of unsophisticated participants, but also on their average stock holdings relative to those held by sophisticated investors. Relative holdings, in turn, are directly related to the relative precision of information about stock payoffs. If unsophisticated investors perceive much more uncertainty about stocks ($\sigma_u^2 >> \sigma_s^2$), they will hold a lower

[19] The literature on the equity premium has recently argued that ρ_s is positive because private equity returns are correlated with the business cycle. See, for example, Heaton and Lucas (2000).

[20] This also explains why, for sophisticated investors, the relevant payoff variance is only the portion that is orthogonal to private returns, $(1-\rho_s^2)\,\sigma_s^2$.

market share. Formally, we have

$$\tilde{\nu}_u = \frac{\nu_u/\sigma_u^2}{\nu_u/\sigma_u^2 + (1-\nu_u)/\sigma_s^2(1-\rho_s^2)}.$$

If information is symmetric and private returns are independent of stock returns, we have $\tilde{\nu}_u = \nu_u$. More generally, $\tilde{\nu}_u$ becomes larger as σ_s^2/σ_u^2 rises and ρ_s^2 falls.

4.3.2 Stock price variation and the business cycle

A simple example shows why the state variables of the business cycle, $\tilde{F}_t^{D,i}$ and $\Delta\tilde{F}_{t-1}^{D,i}$, are typically much more important for equilibrium stock price movements and predictability than the orthogonal off-market factors F_t^B and ΔF_{t-1}^B that change only expected private returns. We conjecture properties for the price function for period $t+1$ that determine payoffs X_{t+1}, and then verify the same properties for the price P_t in (18). Suppose that the future stock price, P_{t+1}, depends positively on the perceived state of the business cycle, $\tilde{F}_t^{Di}$, as well as the perceived change, $\Delta\tilde{F}_t^{Di}$, for $i = u, s$. Suppose also that P_{t+1} depends less, and negatively, on $\tilde{F}_t^{Bi}$, as well as the perceived change $\Delta\tilde{F}_t^{Bi}$, for $i = u, s$.

P_t^i also depends positively on $\tilde{F}_t^{D,i}$ and $\Delta\tilde{F}_t^{D,i}$ because expected payoffs in P_t^i depend positively on $\tilde{F}_t^{D,i}$ and $\Delta\tilde{F}_t^{D,i}$, and these factors are persistent. There are, however, three counteracting effects. The first comes from the risk premium in P_t^u, if there is enough predictability of stock returns in equilibrium. The other two effects occur because a boom today signals higher private returns tomorrow, and it increases the risk premium on local stocks for sophisticated investors (through P_t^s). The risk premium increases via the myopic demand because of the substitutability across assets; it increases via a lower hedging demand because the exposure of sophisticated investors to these state variables—which are positively correlated with stock returns—also increases. Although these counteracting effects exist, it is plausible that there are equilibria in which they are outweighed by the present value effect. This will certainly be true if the number of unsophisticated investors is large enough.

The impact of shocks to $\left(\tilde{F}_t^{B,s}, \Delta\tilde{F}_t^{B,s}\right)$ on prices is limited by the fact that the direct effect offsets the hedging-demand effect on the risk premium. Indeed, an increase in $\tilde{F}_t^{B,s}$ raises the risk premium, since it increases the current expected private return. At the same time, persistence in $\left(\tilde{F}_t^{B,s}, \Delta\tilde{F}_t^{B,s}\right)$

implies that high current values of these variables increase investors' exposure to them in the future. Since they are negatively correlated with stock returns, this increases the hedging demand for stocks and reduces the risk premium. In all our calibrations, the direct effect dominates, so that the negative dependence of prices on these factors is validated. The price coefficients, however, are much smaller than for the business cycle variables. Stock market booms are thus essentially driven by expectations of future cash flows. Essentially all predictability in equilibrium stock returns can be traced to business cycle movements.

4.4 Equilibrium flows and returns

In this section, we decompose equilibrium trades into disagreement and risk-sharing components. We then show how these motives for trade impact key statistics of the joint distribution of flows and returns that are relevant to the stylized facts we are interested in.

4.4.1 Motives for trade: disagreement and risk-sharing

Substituting expression (18) for the equilibrium stock price back into the portfolio-demand formula for unsophisticated investors, we obtain equilibrium flows:

$$\begin{aligned}\Delta\theta_t^{Du} &= \frac{1-\tilde{\nu}}{\tilde{s}\gamma\sigma_n^2}\left(\Delta P_t^u - \Delta P_t^s\right) \\ &= \frac{(1-\tilde{\nu})}{\gamma\sigma_n^2} \\ &\quad \left[\underbrace{\Delta E_t^u X_{t+1} - \Delta E_t^s X_{t+1}}_{\text{Disagreement}} - \underbrace{\rho_s \frac{\sigma_s}{\sigma_b}\Delta E_t^s R_{t+1}^b}_{\text{Segmentation}} - \underbrace{\mathbf{H}^u \Delta\phi_t^u - \mathbf{H}^s \Delta\phi_t^s}_{\text{Hedging demands}}\right].\end{aligned} \tag{19}$$

Trading volume and international equity flows are thus driven by relative changes in the valuations of the two types, captured by the hypothetical prices P_t^u and P_t^s. Differences in valuations arise for two reasons. The first is simply disagreement about future payoffs: unsophisticated investors are net buyers in periods when they become relatively more optimistic than sophisticated investors.

Second, there is trade due to changes in the need for risk-sharing. When sophisticated investors perceive higher expected returns on private opportunities, they prefer to reduce exposure to the business cycle. They thus sell the local asset and unsophisticated investors buy. The effect is not limited to the myopic demand for stocks: the intertemporal hedging demand will typically also change. As discussed above, the key differences in hedging needs across types arise precisely from the presence of private opportunities.

4.4.2 Flow momentum, reversal, and volatility

To examine flow momentum and reversal, we calculate the autocorrelation function of U.S. investors' net purchases of local equities. From (8), these net purchases are proportional to net purchases by unsophisticated investors:

$$\Delta\theta_t^{D,US} = \theta_t^{D,US} - \theta_{t-1}^{D,US} = \nu_{US}\frac{\nu_{uUS} - \nu_u}{1 - \nu_u}\Delta\theta_t^{Du}. \tag{20}$$

The n-th autocorrelation of U.S. net purchases satisfies $\rho_n\left(\Delta\theta_t^{D,US}\right) = \rho_n\left(\Delta\theta_t^{Du}\right) = \rho_n\left(\Delta\theta_t^{Dx}\right)$. The emergence of flow momentum and flow reversal in equilibrium is thus independent of the population parameters: it depends only on the properties of trade across investor types. In other words, the dynamics of U.S. investors' net purchases is characterized by the disagreement and risk-sharing motives.

The composition of investor populations does matter for the volatility of flows, a fact that is used in our calibration strategy. The standard deviation of net purchases is proportional to $|\nu_{uUS} - \nu_u|$, a measure of population heterogeneity:

$$\sigma\left(\Delta\theta_t^{D,US}\right) = \nu_{US}\frac{|\nu_{uUS} - \nu_u|}{1 - \nu_u}\sigma\left(\Delta\theta_t^{Du}\right).$$

In the knife-edge case, where the U.S. population is a scaled version of the total population ($\nu_{uUS} = \nu_u$), holdings of U.S. investors are constant and net flows are zero. Of course, there can still be substantial gross flows if the population of U.S. investors is heterogeneous with respect to investor sophistication.

4.4.3 Return chasing

We examine the relationship between flows and returns in two ways. First, we consider the cross-correlogram of U.S. investors' net purchases and local

returns. As (20) shows, the correlation of flows and returns depends on population parameters only to the extent that they determine which group U.S. investors track:

$$\rho\left(\Delta\theta_t^{D,US}, R_{t-j}^D\right) = sign\left(\nu_{uUS} - \nu_u\right)\rho\left(\Delta\theta_t^{Du}, R_{t-j}^D\right).$$

Whenever $\nu_{uUS} > \nu_u$, there are proportionately more unsophisticated investors in the population of U.S. international investors than in the local population. Holdings and net purchases of U.S. investors are then perfectly correlated with those of unsophisticated investors. In contrast, when $\nu_{uUS} < \nu_u$, U.S. investors track sophisticated investors.

A second way to examine return chasing is to examine the risk premium measured by an econometrician who constructs estimates of expected returns conditional on public information. The econometrician will thus recover $E_t^u R_{t-1}^D$, which can then be related to equilibrium trades (19).

4.4.4 Bursts of gross trading activity

To determine properties of U.S. investors' gross trading activity, it is helpful to first calculate moments of aggregate trading volume in the local market. A natural measure of volume is the turnover of shares. Since every trade is an exchange of shares between the two types of investors, we can define trading volume as:

$$Vol_t := \nu_u\left|\Delta\theta_t^{Du}\right| = (1-\nu_u)\left|\Delta\theta_t^{Ds}\right|.$$

With normally distributed holdings, there are closed-form expressions for the mean and standard deviation of volume, $E\left(Vol_t\right) = \nu_u\sqrt{2/\pi}\sigma\left(\Delta\theta_t^{Du}\right)$ and $\sigma\left(Vol_t\right) = \sqrt{\frac{\pi-2}{2}}E\left(Vol_t\right)$.

Gross purchases by U.S. investors in period t are determined by the type of investor that is a net buyer during the period. Let $\mathbf{1}_{\Delta\theta_t^{Du}>0}$ denote the indicator function for the event that unsophisticated investors are net buyers: that is, $\Delta\theta_t^{Du} > 0$. Mean gross purchases by U.S. investors are given by

$$\begin{aligned} E\left(GP_t^{US}\right) &= \nu_{tUS}E\left[\mathbf{1}_{\Delta\theta_t^{Du}>0}\nu_{uUS}\Delta\theta_t^{Du} - \mathbf{1}_{\Delta\theta_t^{Du}<0}\left(1-\nu_{uUS}\right)\Delta\theta_t^{Ds}\right] \\ &= \frac{1}{2}\nu_{tUS}\left(\frac{\nu_{uUS}}{\nu_u} + \frac{1-\nu_{uUS}}{1-\nu_u}\right)E\left(Vol_t\right). \end{aligned}$$

Mean gross purchases are thus proportional to mean volume.[21]

5. Data

In this section, we describe the data and explain how they are compared with model output. We focus on quarterly data from the G-7 countries—apart from the United States, these are Germany, Japan, the United Kingdom, France, Canada, and Italy—over the period 1977Q1 through 2000Q3. We have selected these countries because they best fit the assumptions of our model. First, flows and returns in these countries are likely to be driven by stable economic relationships.[22] In contrast, the ongoing process of liberalization of equity markets in developing countries may lead to capital flows that are driven by changing risk-sharing opportunities or declining transactions costs.[23] In addition, the absence of trading frictions in our model is more at odds with the institutional environment of emerging markets.

5.1 Dividends

We use data on the dividend yield and the price index of Datastream's international stock market indexes, with all variables converted to constant U.S. dollars. Not surprisingly, per-share dividends exhibit a trend. To obtain a stationary forcing process (D_t) for our model, we follow Campbell and Kyle (1993) in removing an exponential trend. This is described in detail in Appendix B, where we show that it is consistent with our normalization of flows, discussed below.

Table 1 reports key first and second moments of detrended dividends. We have chosen units such that the price index in 1977Q1 equals market capitalization. Mean dividends thus reflect the sizes of the different stock markets. Importantly, mean dividends are more than 3.5 standard deviations above zero for all countries except Italy. There is thus no problem with

[21] The model also predicts that mean gross sales are equal to mean gross purchases, since the mean of net purchases is zero.

[22] While there has been some increase in correlation of stock index returns recently, Brooks and Del Negro (2002) argue that this is a temporary phenomenon connected to an "IT bubble," rather than a permanent shift in market structure.

[23] See Bekaert and Harvey (2003) for a survey of emerging-markets finance.

modelling the dividend as normally distributed in levels.[21]

Preliminary specification analysis of the dynamic behaviour of dividends reveals two features. First, the autocorrelation function switches from positive to negative values after 3 to 4 quarters. Second, while the first two partial autocorrelation coefficients are significant for all countries except Canada, all countries exhibit several significant partial autocorrelation coefficients beyond the first two. To accommodate both properties in a parsimonious way, we follow the system (2) to (5) above, and decompose dividends into a persistent cyclical component, captured by an AR(2) process, and a transitory shock:

$$\begin{aligned} D_t &= \bar{D} + F_t^D + \varepsilon_t^D \\ F_t^D &= a_1 F_{t-1}^D + a_2 F_{t-2}^D + \varepsilon_t^{FD}, \end{aligned} \tag{21}$$

where ε_t^D and ε_t^{FD} are uncorrelated i.i.d. sequences of shocks with zero mean and standard deviations, $\sigma_{\varepsilon D}$ and $\sigma_{\varepsilon FD}$, respectively. F_t^D captures the oscillatory behaviour of the correlogram that is typical of variables affected by the business cycle. The presence of the transitory noise, ε_t^D, that cannot be distinguished from the underlying business cycle movement implies that lags longer than 2 quarters are still helpful in forecasting dividends.

To estimate this process, we use the fact that it permits an ARMA(2,2) representation

$$(D_t - \bar{D}) = a_1 (D_{t-1} - \bar{D}) - a_2 (D_{t-2} - \bar{D}) + u_t + \lambda_1 u_{t-1} + \lambda_2 u_{t-2},$$

where u_t is an i.i.d. sequence of shocks with standard deviation σ_u, and where the parameters satisfy a set of non-linear constraints. Details of the estimation procedure are provided in Appendix C, where we also provide expressions for $\sigma_{\varepsilon D}$ and $\sigma_{\varepsilon FD}$ in terms of the ARMA(2,2) parameters. Table 2 lists the estimation results and some properties of the estimated dividend process.

The persistent component is stationary: the roots of the autoregressive polynomial are outside the unit circle. In most countries, the roots are complex, which accounts for oscillations in the correlogram. In addition, the

[21] Strictly speaking, we are assuming that the "true" dividend process follows a truncated normal distribution. The model is thus an approximation. Table 1 shows that this approximation is very sensible.

persistent component has persistent differences. Indeed, the process of differences of F_t^D satisfies

$$\Delta F_t^D = (-a_2)\,\Delta F_{t-1}^D - (1 - a_1 - a_2)\,F_{t-1} + \varepsilon_t^{FD}.$$

For all countries, we have that $0 < (-a_2) < 1$ and that $(1 - a_1 - a_2)$ is a small positive number. Two counteracting effects are thus at work after a shock hits. First, any change in a certain direction leads to more changes in the same direction, although at a decreasing rate, since $(-a_2) < 1$. If this was the only effect, the level F_t^D would be non-stationary. The second term, however, causes mean reversion in the level, pulling F_t^D towards its mean of zero whenever it is positive, and pulling it up when it is negative. In the impulse response of the level, the first effect will dominate early on, before the second effect takes over. The result is a hump-shaped impulse-response function.

The persistent component explains almost all the variation in dividends: its share of total variance is larger than 96 per cent for all countries except Italy. For three of the seven countries, the volatilities of the shocks that hit the persistent component in any given quarter are also higher than those of transitory shocks. Still, changes in dividends are typically less persistent than changes in the persistent component. Changes in dividends can be decomposed into changes in the persistent component, which are positively serially correlated, and changes in the temporary component, which are negatively serially correlated and thus reduce overall persistence.

5.2 Equity flows

5.2.1 Sources

We obtain data on the international equity flows of U.S. investors from the Treasury International Capital (TIC) reporting system of the U.S. Treasury.[25] Financial institutions (banks, bank holding companies, securities brokers, dealers, and non-banking enterprises) must report to the Treasury,

[25] A number of related studies use the same data set: Tesar and Werner (1993, 1995), Bohn and Tesar (1996), Brennan and Cao (1997), and Albuquerque, Bauer, and Schneider (2003). See Froot, O'Connell, and Seasholes (2001) and Levich (1994) for a description of the limitations and advantages of U.S. Treasury data.

each month, by country, on all of their transactions with foreigners in long-term securities (e.g., stocks and bonds), if their aggregate purchases or sales total more than US$2 million in the month. As a result, the Treasury receives comprehensive data on cross-border equity transactions for most U.S. investors. The Treasury collects data by geographic centre and not by the country of origin of the security. This means that the data can be unrepresentative for countries that have large international financial centres, such as the United Kingdom. Warnock and Cleaver (2002) examine the TIC data in detail and find that transactions to the United Kingdom are overstated while transactions to other countries are understated. A typical example of this is the purchase by U.S. investors of stock from, say, an Italian company issuing securities in the euro-equity market through banks in London, which is recorded as a sale of U.K. equity.

Data on the volume of trading are from Datastream's Global Equity Indices and give the aggregation of the number of shares traded multiplied by the closing price for each stock. Finally, we obtain data on equity holdings from a joint report issued by the U.S. Treasury, Federal Reserve Bank of New York, and Board of Governors of the Federal Reserve System (2003). This report is based on TIC data and the 1997 benchmark survey of U.S. investors.

5.2.2 Matching model and data flows

Flow and volume data record sums over all transactions in a given month or quarter; the TIC database does not provide guidance on which days, and hence at what prices, the transactions took place. In contrast, our discrete-time model makes predictions about holdings at a point in time. To match model-implied changes in holdings to flow data, we need to normalize the latter. One convenient way to do this is to divide flows by total market capitalization at the beginning of the period. To see why this makes sense, suppose that there are n dates between t and $t-1$ at which transactions are recorded. Let x_i denote the fraction of the net change in U.S. investors' holdings, $\theta_t^{DUS} - \theta_{t-1}^{DUS}$ (with θ_t^{DUS} measured as a fraction of outstanding shares), that takes place at date t_i. Then, normalized net flows are given by

$$NF_t = \frac{1}{P_t^*}\left(\theta_t^{DUS} - \theta_{t-1}^{DUS}\right)\sum_{i=1}^{n} x_i P_{t_i}^* = \left(\theta_t^{DUS} - \theta_{t-1}^{DUS}\right)\sum_{i=1}^{n} x_i \frac{P_{t_i}^*}{P_t^*},$$

where P_t^* is the undetrended local stock price.[26]

Normalized flows are thus equal to the change in holdings multiplied by a weighted average of within-month capital gains. In what follows, we match normalized net flows to the first term, $(\theta_t^{DI,S} - \theta_{t-1}^{DI,S})$. This match is exact if all transactions take place on the first day of the month; that is, $t_1 = t$, $x_1 = 1$, and $x_i = 0$ for $i > 1$. Some evidence on the importance of the resulting bias can be obtained by comparing results with the polar-opposite case, when flows are normalized by the end-of-period market capitalization (i.e., $t_n = t + 1$ and $x_n = 1$). In terms of our stylized facts, this change somewhat reduces both the contemporaneous correlation of flows and returns and the persistence of flows, but the effect is on the order of a few percentage points for all countries. We conclude that the normalization is reasonable.

It is well known that turnover (that is, the ratio of trading volume to market capitalization) exhibits an increasing trend. Not surprisingly, the same is true for gross flows to and from all countries in our model, after they have been normalized by market capitalization. Our model does not allow for this type of trend in trading activity: equilibrium holdings, and hence their differences, are stationary. However, this need not affect the model's relevance for stylized facts about *net* flows. Indeed, it is plausible that much of the trend in trading activity is due to features of the trading process that have been simplified away in the model, but that are not germane to the behaviour of net flows.[27]

Of course, if our model is correct, not all of the gross flows are unrelated to movements in net flow. In our calibration, we thus insist on obtaining moments for our model-implied stationary turnover series that approximate values observed in the data. In particular, we calibrate the expected value of

[26] Appendix B shows that this normalization is consistent with exponential detrending of dividend levels.

[27] First, the actual population of U.S. investors does not consist of long-lived agents that do not have any idiosyncratic liquidity needs. Trades due to finite investment horizons or other liquidity reasons need not affect net flows as long as they average out across investors. They will, however, be recorded as gross flows and volume. Since their frequency is arguably increasing with the increase in market participation, this might account for the trend in the gross measures. A second plausible reason for the trend is rebalancing across different securities. We assume throughout that there is a single (index) security that all investors hold. In fact, there are many stocks, and agents who hold an index rebalance as market weights change. Rebalancing does not add to net flows, but it is recorded as gross flows. It is also likely more frequent because share repurchases and issues have become more common in recent years.

turnover to the average turnover over the years 1995–2000. We then compare other model moments with similar long-run averages from the data.

5.2.3 Summary statistics

Table 3a presents summary statistics for net purchases of stocks abroad by U.S. investors, as well as excess returns on domestic indexes for the countries we consider. The mean excess returns in the table are based on detrended data, which means that the effects of dividend growth are already removed. This explains why excess returns are smaller than the mean equity premiums usually reported from raw data, and why Sharpe ratios implied by the table are unusually low. In our set of countries, changes in American investors' holdings are small relative to total market capitalization. Within a given quarter, it is rare to see a change in position of more than 1 per cent of market capitalization.

Table 3a documents two key stylized facts about the joint distribution of net inflows and excess returns. First, net inflows are persistent. The first autocorrelation coefficient ranges between 0.16 for Switzerland and 0.52 for Canada. In all countries but Italy, it is statistically significant at the 5 per cent level. The persistence of net inflows is not due to trends. Figure 1 plots the net inflow series for all countries in our model. It is apparent that the main feature is slow transitions from periods of high to low net inflows.[28] For example, American investors pulled money out of the French stock market in the late 1970s and reinvested it there again in the mid-1980s. They did essentially the opposite in the Netherlands: positions that were built slowly over the late 1970s and early 1980s were unwound between 1983 and 1986. The second stylized fact is that the contemporaneous correlation between domestic excess stock returns (measured in U.S. dollars) and net inflows from the United States is strongly positive.

Table 3b reports summary statistics for holdings, gross flows, and volume. U.S. investors hold significant fractions of the market in all of the countries in our sample, except Italy. Gross purchases and sales are of the same order of magnitude in all the countries. The stylized fact that gross sales and

[28] For some countries, such as Germany and Italy, there is a marked change in volatility between the late 1970s and early 1980s and more recent years. This reflects an increase in overall trading activity. This effect, however, does not induce a trend in the mean of net inflows.

purchases are highly positively correlated holds both in the time series for every country and in the cross section of countries. Importantly, the time-series results reflect not only trend behaviour. While there are trends in gross flows over the whole sample, behaviour over a 5-year period is driven mostly by volatility that is common to both series. Figure 2 illustrates this for the countries in our sample. Finally, volume, measured by the value of all trades divided by market capitalization, varies widely across countries. It is interesting, however, that holdings of U.S. investors appear to turn over *less* frequently than holdings of other investors within the country. This is true for all but two of the countries in our sample: Canada and the United Kingdom. This fact will be of interest in our calibration in section 6.

6. Quantitative Analysis

In this section, we first describe how we calibrate the model to dividend and flow data. The procedure outlined in section 6.1 applies to all countries in our sample. We then provide some further model statistics not used in the calibration and compare them with the data. We use structural impulse responses and variance decomposition analysis to interpret our findings.

6.1 Calibration

6.1.1 Preferences

One period in the model corresponds to 1 quarter. We choose an annual discount rate of 4 per cent; that is, $\beta = 0.9901$. The coefficient of absolute risk aversion is set to $\gamma = 10$.

6.1.2 Investment opportunities

Local dividends and world asset returns are taken directly from the data. For dividends, we use the detrended process estimated in section 5.1. From (4), we assume that the world return is unpredictable and uncorrelated with local dividends. Its mean and standard deviation are matched to the U.S. stock market return: $\bar{R}^W = 0.0187$ and $\sigma_W = 0.074$.

It is difficult to construct an observable counterpart of the returns on private investment opportunities. Our strategy is to first impose a number

of a priori plausible restrictions that give rise to a two-parameter family of processes, with the free parameters η_D and η_B introduced in (3). We then fix the remaining parameters to match selected moments on stock market trading activity. We impose throughout that the unconditional mean and variance of private returns are the same as those of the return on the world asset. In addition, we allow for three specific features of private returns.

First, private returns can be predictable. Predictability has been documented in many securities markets and it is certainly prevalent for non-traded assets, where returns do not need to be competed away quickly. Second, both the predictable and the unpredictable components of returns may be correlated with the local business cycle. In our model, the latter is captured by dividends. Third, there may be persistent factors other than the local business cycle that affect expected private returns. This feature is of interest because some opportunities chased by sophisticated investors active in the local markets may in fact be located in other countries.

According to (3), the first component of private expected returns is proportional to the persistent component of local dividends, F_t^D. The second component is driven by a process, F_t^B, that is independent of F_t^D and also has an AR(2) structure. We impose that it captures oscillations at business cycle frequencies by setting the AR(2) parameters equal to those of the persistent component in U.S. dividends. As a normalization, the variance of shocks to F_t^B is set equal to that of F_t^D. The overall volatility of expected returns and the relative importance of the local business cycle is then governed by the parameters η_D and η_Z.

In our baseline calibration, we also fix $\rho\left(z^B, z^D\right) = 0.5$ and $\sigma_{\eta}^2 = 0.1$. Sensitivity analysis has shown that the performance of the model does not depend strongly on these values. Once they are fixed, and given values for η_D and η_Z, the variance of unexpected returns, $\sigma_{\varepsilon B}^2$, must be chosen to ensure that the unconditional variance of private returns matches that of the world asset return. Our specification of investment opportunities thus leaves two degrees of freedom that can be used to match statistics of trading activity.[29]

6.1.3 Matching flow moments

In total, we are left to choose five parameters: the fractions ν_u, ν_{US}, and ν_{nUS}, that govern the composition of the investor population, and the

[29] This assumption is not really restrictive, since F_t^B is not directly linked to observables. It could simply be interpreted as sophisticated investors' *perceived* expected returns.

numbers η_D and η_Z that govern the volatility and business cycle correlation of private returns. We select these parameters to best match five moments of trading activity: mean volume, mean local holdings, and mean gross purchases by U.S. investors, as well as the standard deviation and the first autocorrelation of net purchases by U.S. investors. In addition, we use the positive sign of the contemporaneous correlation of U.S. net purchases and returns to provide guidance on which investor type is more prevalent in the U.S. investor population. The relevant model statistics are defined in section 4.4 and their observable counterparts are explained in section 5.2.

Table 4 lists the parameter values of the baseline calibration for all countries in our model together with data and model values of the target moments. The target moments are matched tightly, although the model understates mean volume in Germany, Japan, Italy, and the United Kingdom. The parameter values for the expected off-market return process are similar across countries. The business cycle component is most important in Italy, the country where the persistent component accounts for less of the dividend variance (cf. Table 2). In contrast, the independent component, F_t^B, plays a larger role in driving private returns available to investors in the Canadian stock market. This is needed in order to increase trading volume (see (19)).

For Japan, Italy, and the United Kingdom, the model generates small volatility of unsophisticated investors' flows, which brings down the volatility of trading (see section 4.4). Nonetheless, the model can still match the volatility of U.S. investors' flows as long as ν_{uUS} is sufficiently larger than ν_u (see (20)). For Germany, the model generates an average trading volume comparable with other countries, but the data indicate a much larger volume. In contrast, the model performs well in predicting mean gross purchases in these markets. The lone exception is the United Kingdom, possibly because it is a large international financial centre (see Levich 1994).

With the exception of Japan and the United Kingdom, the average international U.S. investor is sophisticated: $\nu_{uUS} < 0.5$. However, for all countries, $\nu_u < \nu_{uUS}$, which means that the average U.S. international investor is less sophisticated than the average local investor. Being relatively less sophisticated means that the aggregate net flows of U.S. investors are proportional to unsophisticated investors' net flows (see (20)). This fact is consistent with the view that U.S. investors have worse private information than local investors, usually associated with the existence of a home bias. Importantly, Table 4 indicates that cross-country heterogeneity observed in the difference, $\nu_u - \nu_{uUS}$, is not as significant as within-country heterogene-

ity measured by $\nu_{ij} - 0.5$. Trading is thus not motivated by differences in population across countries, but by differences in investor populations within countries, as would be expected in G-7 economies.

6.2 Further predictions for flows and returns

Of the four stylized facts we set out to explain, only the persistence of net flows (as reflected in $corr\left(\Delta\theta_t^n, \Delta\theta_{t-1}^n\right)$) is directly used to calibrate the model. Table 5 reports further data and model statistics not used in the calibration that are relevant to the other stylized facts. In addition, Figures 3 and 4 present graphs of the entire cross-correlogram of returns and flows and the autocorrelogram of flows for the six countries in our sample.

6.2.1 Simultaneous buying and selling

The model produces high positive contemporaneous correlation between gross purchases and gross sales. Gross trading activity of U.S. investors thus occurs in bursts of simultaneous buying and selling. The fact that we overpredict these bursts of trading could be due to transitory idiosyncratic shocks that are recorded as gross flows. The United Kingdom and Italy are the only two countries for which the model predicts a negative correlation between purchases and sales of local stocks by U.S. investors.

6.2.2 Flow continuation and flow reversal

The first column in Figures 3 and 4 shows the autocorrelogram of U.S. investors' net purchases, (equivalently, that of unsophisticated investors' net purchases), with 90 per cent confidence bands computed with Newey-West errors. It is remarkable how well the model captures the J-curve pattern evident in the data. The J-curve pattern displays flow continuation up to 3 (and sometimes 4) lags and flow reversal at lags 5 and 6. The data also display a cyclical pattern, with the flow correlations increasing again after lag 6. This is also captured in the model—as a virtue of the AR(2) processes estimated for dividends—though at longer horizons. Only the United Kingdom and Japan display significantly more persistence, in the short run, in the data than in the model.

6.2.3 Return chasing

Return chasing behaviour is apparent both in Table 5 and in the cross-correlograms in the second column of Figures 3 and 4. The model somewhat overpredicts the contemporaneous correlation of returns and net purchases for France, the United Kingdom, and Italy, whereas the performance for Canada, Germany, and Japan is quite satisfactory. Moreover, the model captures the tent-shaped curve around the contemporaneous correlation displayed in the data. The model matches well the significant return chasing in France and Germany, and the absence of it in Italy. The model misses the correlation of lagged returns and current flows for the United Kingdom, Canada, and Japan. However, it captures the qualitative nature of cyclicality in the correlation of lagged returns and flows: it is low and negative at 2 and 3 lags, and increases after 4 or 5 lags.

The model also generates positive correlation between net purchases by U.S. investors and expected returns based on public information:

$$\rho\left(\Delta\theta_t^{D,US}, E_t^u\left(R_{t+1}^D\right)\right) > 0.$$

This is consistent with evidence presented by Bohn and Tesar (1996) for the countries in our sample. Bohn and Tesar estimate expected returns using a comprehensive set of instruments that proxy the public information set. They then show that U.S. investors move into a market when their fitted expected returns are high.

6.2.4 Other statistics

The value of $E\left(Vol_t\right)$ is calibrated to the data, which means that, in our model, $\sigma\left(Vol_t\right) = 0.7(5) \times E\left(Vol_t\right)$. The model predicts that $E\left(Vol_t\right) > \sigma\left(Vol_t\right)$, which is robust across all countries. The exact quantitative performance of the model varies considerably across countries. The model does well for Germany, but overpredicts the volatility of trading volume by a factor of 3 for France and Canada. The model significantly underpredicts volatility in trading volume for Italy (see Table 4). With the exception of Japan, current flows predict one-quarter-ahead returns both in the data and in the model. For Japan, both the data and the model display a negative correlation between flows and future returns.

The model exhibits both an equity premium puzzle and a volatility puzzle for price levels (not documented), which are two common weaknesses of

macroeconomic asset-pricing models discussed in detail by Campbell (2000). These results are not entirely surprising, since, for technical reasons, our model features constant discount rates. The frictions we introduce thus cannot produce highly amplified effects on price levels.

6.3 Interpretation

To provide intuition for our numerical results, we discuss the role of various structural shocks in generating the stylized facts we are interested in. As a representative example, we focus on the French stock market.

6.3.1 Impulse responses and variance decomposition

An impulse-response function describes the dynamic response of equilibrium prices and trades to a one-time structural shock. We normalize the size of the shock to one standard deviation. Impulse-response functions are easily calculated from the model's stationary vector autoregressive representation.[30] Figures 5-7 plot the model's response to an innovation to the persistent component of local dividends (the "business cycle shock," ε_t^{FD}), to a transitory shock to dividends, ε_t^{D}, and to an innovation to the off-market factor, ε_t^{FB}, respectively. Each row corresponds to a variable or group of variables: from top to bottom, we have the local stock price, P_t, the forecast errors on the business cycle by both investor types, $F_t^D - \hat{F}_t^{Ds}$ and $F_t^D - \hat{F}_t^{Du}$ (plotted together in the second row), the local per-dollar stock return, unsophisticated investors' net purchases, and conditional one-quarter-ahead forecasts of the local stock return.

Not all of the structural shocks discussed above are equally important for a given model statistic. To quantify the role of the different shocks, we provide variance decompositions of key second moments. Figure 8 plots the contribution of every shock to the covariance of unsophisticated investors' flows and returns, the covariance of unsophisticated investors' current and lagged flows, the covariance of unsophisticated investors' flows and their expected returns (the return chasing effect), and the covariance of current and

[30] Let $\mathbf{x}_t = (F_t^D, F_{t-1}^D, \Phi_t^*, D_t, R_t^W, R_t^B)'$. It can be verified that the vector $\mathbf{x}_t$ has a first-order vector autoregressive representation where the errors are the economy's structural shocks, and that this characterization of $\mathbf{x}_t$ fully describes the equilibrium of the model. Any variable in the economy, such as asset holdings and flows or realized and expected returns, can be easily constructed from $\mathbf{x}_t$.

lagged stock returns.[31]

6.3.2 Return chasing and the business cycle

Persistent local business cycle shocks induce return chasing. The variance decompositions show that ε_t^{FD} accounts for most of the correlation of flows with both current and past returns. While temporary dividend shocks contribute to the contemporaneous correlation, they actually have a small negative effect on the lagged correlation. Shocks to the off-market factor are largely irrelevant for return chasing.

The impulse response to a positive innovation, ε_t^{FD}, is shown in the first column of Figure 5. On impact, prices increase in response to higher current dividends and future expected payouts. Unsophisticated investors observe these public signals. They underreact to the shock, since they cannot be sure that F_t^D has actually moved. Sophisticated investors underreact by less, because they have more signals to rely on. Disagreement trading by itself would thus lead the more optimistic sophisticated investors to buy shares. Improved private opportunities also trigger risk-sharing trades. Both the myopic and the hedging demands of sophisticated investors decrease as they try to get rid of tradable business cycle risk. Overall, the risk-sharing effect dominates: sophisticated investors sell the domestic stock market as prices rise, which contributes to positive contemporaneous correlation of net purchases and returns.

The high stock return that occurs on impact is followed by further net purchases by unsophisticated investors, before reversal sets in. In fact, for about three quarters after the impact effect, disagreement and risk-sharing trades move in the same direction, generating pronounced return chasing. On the one hand, disagreement is reduced as unsophisticated investors learn the nature of the shock. This encourages them to buy. On the other hand, business cycle momentum creates more private opportunities. Sophisticated investors' incentive to sell shares thus also increases, at least in the short run. After about 3 quarters, reversal sets in. The disagreement effect weakens as unsophisticated investors have learned the nature of the shock. At the same time, the return on private opportunities begins to revert to the mean. As a result, sophisticated investors return to the stock market. Importantly, both return chasing and the eventual reversal are predictable consequences of the

[31] We omit the shocks to sophisticated investors' private signals and to transitory off-market returns, because they have a minimal direct contribution to these moments.

initial shock (and concomitant high return). This effect thus explains the observed oscillations in the cross-correlogram.

Transitory shocks to dividends contribute only to a contemporaneous correlation of flows and returns. In response to such a shock (shown in Figure 6), both types of investors see dividends increase and assign positive probability to the shock being persistent. Unsophisticated investors, however, become more optimistic than sophisticated investors, because they have fewer signals about the persistent component, F_t^D. They expect a continuation of high prices and future positive returns, and buy the local stock market. In contrast, sophisticated investors are less optimistic and sell the local stock market. The impact of the shock induces a positive correlation between unsophisticated investor flows and returns. After the impact, trades driven by a transitory shock are quickly reversed as investors correct their forecast errors. Too large a contribution from these shocks would thus prevent the model from matching the positive correlation of net purchases with lagged returns.

6.3.3 Flow momentum, reversal, and risk-sharing

The autocorrelation of flows is mainly driven by both business cycle shocks and shocks to the off-market factor. For both types of shock, the major motive of trade is risk-sharing: as off-market opportunities improve, sophisticated investors try to shed tradable business cycle risk to load up on non-tradable risk. The impulse response of flows to the two shocks is thus similar in shape.

Initially, there is a fair amount of disagreement: unsophisticated investors underestimate the actual state of the business cycle. Whereas this is due to underreaction after an F_t^D shock (Figure 7), it is due to *overreaction* after an F_t^B shock (see Figure 5). In the latter case, unsophisticated investors see only a drop in prices, which they will partly attribute to a downturn in the local business cycle and partly to the F_t^B shock. Since prices must fall on impact to entice unsophisticated investors to buy, the F_t^B contributes negatively to the contemporaneous correlation of flows and returns.

Disagreement makes it costly for sophisticated investors to sell early on. As the shock persists, investors learn the nature of the shock and the forecast error is reduced. Sophisticated investors keep leaving the stock market, generating persistent flows. Importantly, only persistent shocks are able to generate persistence in flows and returns. Transitory shocks to dividends

produce very quick reversals of flows that translate into negatively serially correlated flows. This constrains the model's ability to generate the observed trading patterns: calibrations that create a bigger role for transitory shocks improve the model's performance in generating a positive $cor\left(\Delta\theta_t^a, R_t^D\right)$, but worsen the model's performance in terms of flow persistence.

Bibliography

Admati, A.R. 1985. "A Noisy Rational Expectations Equilibrium for Multi-Asset Securities Markets." *Econometrica* 53: 629–57.

Albuquerque, R., G. Bauer, and M. Schneider. 2003. "Characterizing Asymmetric Information in International Equity Markets." University of Rochester Working Paper.

Bailey, W. and C.X. Mao. 2001. "Investment Restrictions and the Cross-Border Flow of Information: Some Empirical Evidence." Cornell University. Photocopy.

Bekaert, G. and C. Harvey. 2003. "Emerging Markets Finance." *Journal of Empirical Finance* 10: 3–55.

Bohn, H. and L. Tesar. 1995. "The US Investment Portfolio and Mean-Variance Optimization." University of California at Santa Barbara Working Paper.

———. 1996. "US Equity Investment in Foreign Markets: Portfolio Rebalancing or Return Chasing?" *American Economic Review Papers and Proceedings* 86: 77–81.

Brennan, M.J. and H.H. Cao. 1997. "International Portfolio Investment Flows." *Journal of Finance* 52: 1851–80.

Brooks, R. and M. Del Negro. 2002. "The Rise in Comovement across National Stock Markets: Market Integration or IT Bubble?" Federal Reserve Bank of Atlanta. Photocopy.

Campbell, J.Y. 2000. "Asset Pricing at the Millennium." *Journal of Finance* 55: 1515–67.

Campbell, J. and A.S. Kyle. 1993. "Smart Money, Noise Trading and Stock Price Behaviour." *Review of Economic Studies* 60: 1–34.

Campbell, J., S.J. Grossman, and J. Wang. 1993. "Trading Volume and Serial Correlation in Stock Returns." *Quarterly Journal of Economics* 108:905–39.

Choe, H., B.-C. Kho, and R.M. Stulz. 1999. "Do Foreign Investors Destabilize Stock Markets? The Korean Experience in 1997." *Journal of Financial Economics* 54: 227–64.

———. 2001. "Do Domestic Investors Have More Valuable Information About Individual Stocks Than Foreign Investors?" NBER Working Paper No. 8073.

Coval, J. 1999. "International Capital Flows when Investors have Local Information." University of Michigan. Photocopy.

Dumas, B., C. Harvey, and P. Ruiz. 2002. "Are Correlations of Stock Returns Justified by Subsequent Changes in National Outputs?" Duke University. Photocopy.

Frankel, J.A. and S. Schmukler. 1996. "Country Fund Discounts and the Mexican Crisis of December 1994: Did Local Residents Turn Pessimistic before International Investors?" *Open Economies Review* 7: 511–34.

Froot, K.A. and T. Ramadorai. 2001. "The Information Content of International Portfolio Flows." NBER Working Paper No. 8472.

Froot, K.A. and J. Tjornhom. 2002. "Decomposing the Persistence of International Equity Flows." NBER Working Paper No. 9079.

Froot, K.A., P. O'Connell, and M. Seasholes. 2001. "The Portfolio Flows of International Investors." *Journal of Financial Economics* 59: 151–94.

Griffin, J.M., F. Nardari, and R.M. Stulz. 2002. "Daily Cross-Border Equity Flows: Pushed or Pulled?" Arizona State University. Photocopy.

Grinblatt, M. and M. Keloharju. 2000. "The Investment Behaviour and Performance of Various Investor Types: A Study of Finland's Unique Data Set." *Journal of Financial Economics* 55: 43–67.

Hamao, Y. and J. Mei. 2001. "Living with the "Enemy": An Analysis of Foreign Investment in the Japanese Equity Market." *Journal of International Money and Finance* 20: 715–35.

Hau, H. 2001. "Location Matters: An Examination of Trading Profits." *Journal of Finance* 56: 1959–83.

Hau, H. and H. Rey. 2002. "Order Flows, Exchange Rates and Asset Prices." INSEAD Working Paper.

Heaton, J. and D. Lucas. 2000. "Asset Pricing and Portfolio Choice: The Role of Entrepreneurial Risk." *Journal of Finance* 55: 1163–98.

Karolyi, G.A. 1999. "Did the Asian Financial Crisis Scare Foreign Investors Out of Japan?" Ohio State University. Photocopy.

Levich, R. 1994. "Comment on International Equity Transactions and U.S. Portfolio Choice." In *The Internationalization of Equity Markets*, edited by J. Frankel. 221–27. University of Chicago Press.

Lewis, K.K. 1999. "Trying to Explain Home Bias in Equities and Consumption." *Journal of Economic Literature* 37: 571–608.

Merton, R. 1987. "A Simple Model of Capital Market Equilibrium with Incomplete Information." *Journal of Finance* XLII: 483–510.

Portes, R. and H. Rey. 2000. "The Determinants of Cross-Border Equity Flows: The Geography of Information." London Business School. Photocopy.

Seasholes, M. 2000. "Smart Foreign Traders in Emerging Markets." Harvard University. Photocopy.

Shapiro, A. 2002. "The Investor Recognition Hypothesis in a Dynamic General Equilibrium: Theory and Evidence." *Review of Financial Studies* 15: 97–141.

Stulz, R. 1999. "International Portfolio Flows and Security Markets." In *International Capital Flows*, edited by M. Feldstein. Chicago: University of Chicago Press.

Tesar, L. and I. Werner. 1993. "International Equity Transactions and U.S. Portfolio Choice." In *The Internationalization of Equity Markets*, edited by J.A. Frankel. Chicago: The University of Chicago Press.

———. 1995. "Home Bias and High Turnover." *Journal of International Money and Finance* 4: 467–92.

U.S. Treasury, Federal Reserve Bank of New York, and Board of Governors of the Federal Reserve System. 2003. "Report on Foreign Portfolio Holdings." Available at http://www.treas.gov/tic/fpis.html.

Wang, J. 1994. "A Model of Competitive Stock Trading Volume." *Journal of Political Economy* 102: 127–68.

Warnock, F.E. and C. Cleaver. 2002. "Financial Centers and the Geography of Capital Flows." Board of Governors of the Federal Reserve System International Finance Discussion Paper No. 722.

Table 1: Summary Statistics of Dividends

	μ	σ	ρ_1
Canada	4.89	0.34	0.93
France	2.19	0.47	0.96
Germany	5.50	1.41	0.97
Italy	0.57	0.27	0.98
Japan	14.81	2.87	0.98
U.K.	12.91	2.23	0.92
U.S.	91.51	3.61	0.90

Note: Mean (μ), standard deviation (σ), and first autocorrelations (ρ_1) of detrended, seasonally adjusted dividends, deflated by U.S. CPI: 1977Q1–2000Q3.

Table 2: Estimated Dividend Process

	a_1	a_2	σ_ε^{FD}	σ_ε^{D}	$\sigma(F_t^D)$	$p_1(\Delta F_t^D)$	Roots	$p_1(\Delta D_t)$
Canada	1.859	-0.896	0.036	0.073	0.409	0.88	1.04 ± 0.21	-0.002
France	1.369	-0.420	0.110	0.026	0.458	0.39	1.11, 2.15	0.327
Germany	1.734	-0.773	0.143	0.183	1.095	0.75	1.12 ± 0.19i	0.217
Italy	1.685	-0.708	0.031	0.170	0.272	0.70	1.12, 1.26	0.454
Japan	1.212	-0.275	0.783	0.031	2.988	0.24	1.10, 3.30	0.248
U.K.	1.223	-0.294	0.572	0.031	2.403	0.26	1.12, 3.04	0.252
U.S.	1.679	-0.747	0.698	0.818	3.836	0.71	1.12 ± 0.28i	0.088

Note: "Roots" refers to the roots of the autoregressive polynomial that gives the law of motion of F_t^D.

Table 3a: Excess Returns and Net Flows

	Excess returns (%)		Net inflows (%)			
	$\mu(R^D)$	$\sigma(R^D)$	$\mu(NF^*)$	$\sigma(NF^*)$	$\rho_1(NF^*)$	$\rho(R^D, NF^*)$
Canada	1.4	8.1	0.17	0.37	0.52	0.27
France	1.5	11.7	0.15	0.28	0.46	0.17
Germany	0.7	9.8	0.03	0.14	0.35	0.28
Italy	0.9	15.0	0.05	0.31	0.16	0.13
Japan	0.8	13.0	0.05	0.13	0.45	0.40
U.K.	0.8	9.3	0.12	0.24	0.51	0.16
U.S.	1.9	7.4	-	-	-	-

Notes: Means (μ), standard deviations (σ), and first autocorrelations (ρ_1) for excess returns (log quarterly US$ returns minus 3-month T-bill rate) and net inflows (net purchases of foreign stocks by U.S. investors, normalized by beginning-of-period market capitalization). $\rho(R^D, NF^*)$ is the contemporaneous correlation coefficient of excess returns and net inflows. Quarterly data, 1977Q2–2000Q3.

Table 3b: Holdings, Turnover, and Gross Flows

	U.S. holdings (%)	Volume (%)		Gross flows (%)		
	h^{US}	Mean	Std. dev.	$E(GP^{US})$	$E(GS^{US})$	$\rho(GP^{US}, GS^{US})$
Canada	14.3	14.1	2.7	3.2	3.0	0.97
France	12.7	16.0	3.4	0.9	0.9	0.62
Germany	9.9	51.6	13.2	1.0	1.0	0.87
Italy	1.1	86.2	35.1	0.8	0.8	0.60
Japan	39.0	4.8	2.7	0.9	0.8	0.91
U.K.	12.4	3.9	2.1	4.5	4.5	0.95
U.S.	14.3	14.1	2.7	3.2	3.0	0.97

Note: U.S. holdings are a fraction of local market capitalization, as of 31 December 1999. Volume is total value of shares traded divided by market capitalization. Gross purchases (GP) and gross sales (GS) are divided by market capitalization. All gross flow and volume statistics are averages over 1995Q1–2000Q3.

Table 4. Parameters and Calibrated Moments

	France		Canada		Germany		U.K.		Japan		Italy	
Parameters												
Private returns	η_D	η_B	η_D	η_B	η_D	η_B	η_D	η_B	η_D	η_B	η_D	η_B
	0.085	0.087	0.070	0.294	0.045	0.054	1e-4	0.011	0.001	0.010	0.110	0.120
No. of unsophisticated		ν_U		ν_U		ν_U		ν_U		ν_U		ν_U
		0.40		0.38		0.35		0.40		0.80		0.01
U.S. population	ν^*	ν^*_U	ν^*	ν^*_U	ν^*	ν^*_U	ν^*	ν^*_U	ν^*	ν^*_U	ν^*	ν^*_U
	0.124	0.43	0.14	0.41	0.10	0.37	0.12	0.80	0.39	0.84	0.01	0.40
Moments	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
$\mu(\theta_t^{DTS})$ in %	12.7	12.7	14.3	14.3	9.9	9.9	12.4	12.4	39.0	39.0	1.1	1.1
$\sigma(\Delta\theta_t^{DTS})$ in %	0.28	0.28	0.37	0.37	0.14	0.14	0.24	0.24	0.13	0.13	0.31	0.31
$\rho_1(\Delta\theta_t^{DTS})$	0.46	0.46	0.52	0.52	0.35	0.35	0.51	0.32	0.45	0.25	0.16	0.16
$\mu(Vol_t)$ in %	16.9	16.7	14.1	14.1	51.6	12.3	3.9	0.95	4.8	1.0	86.2	1.0
$\mu(GP_t^{DS})$ in %	0.9	2.1	3.2	2.0	1.0	1.2	4.5	0.13	0.9	0.4	0.8	0.13

Table 5: Non-Calibrated Moments

	France		Canada		Germany	
	Data	Model	Data	Model	Data	Model
$\text{Corr}(NF_t^{US}, R_t^D)$	0.17	0.37	0.27	0.14	0.28	0.27
$\text{Corr}(NF_t^{US}, E_t^u(R_{t-1}^D))^{(*)}$	+	0.15	+	0.17	+	0.20
$\sigma(Vol_t)$ in %	3.1	12.6	2.7	10.7	13.2	9.3
$\text{Corr}(GP_t^{US}, GS_t^{US})$	0.63	0.98	0.97	0.97	0.87	0.99

	U.K.		Japan		Italy	
	Data	Model	Data	Model	Data	Model
$\text{Corr}(NF_t^{US}, R_t^D)$	0.16	0.28	0.40	0.42	0.13	0.64
$\text{Corr}(NF_t^{US}, E_t^u(R_{t-1}^D))^{(*)}$	+	0.21	+	0.26	+	0.20
$\sigma(Vol_t)$ in %	2.1	0.7	2.7	0.8	35.1	0.8
$\text{Corr}(GP_t^{US}, GS_t^{US})$	0.95	-0.17	0.91	0.89	0.60	-0.44

Note: Data for $\text{Corr}(NF_t^{US}, R_t^D)$ are taken from Table 2 in Bohn and Tesar (1996).

Figure 1: Net Purchases by U.S. Investors as a Fraction of Local Market Capitalization

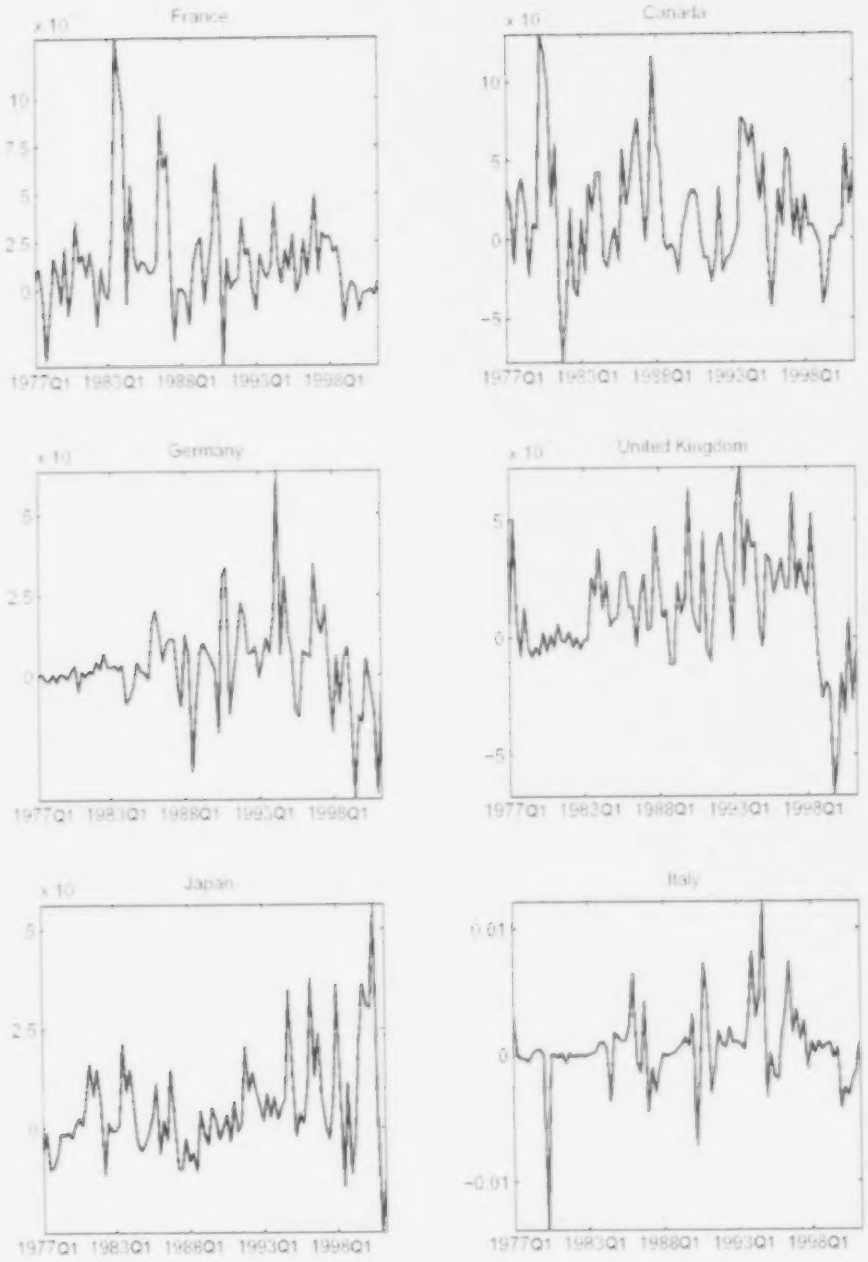


Figure 2: U.S. Investors Gross Purchases and Sales of Foreign Equities as a Fraction of Local Market Capitalization

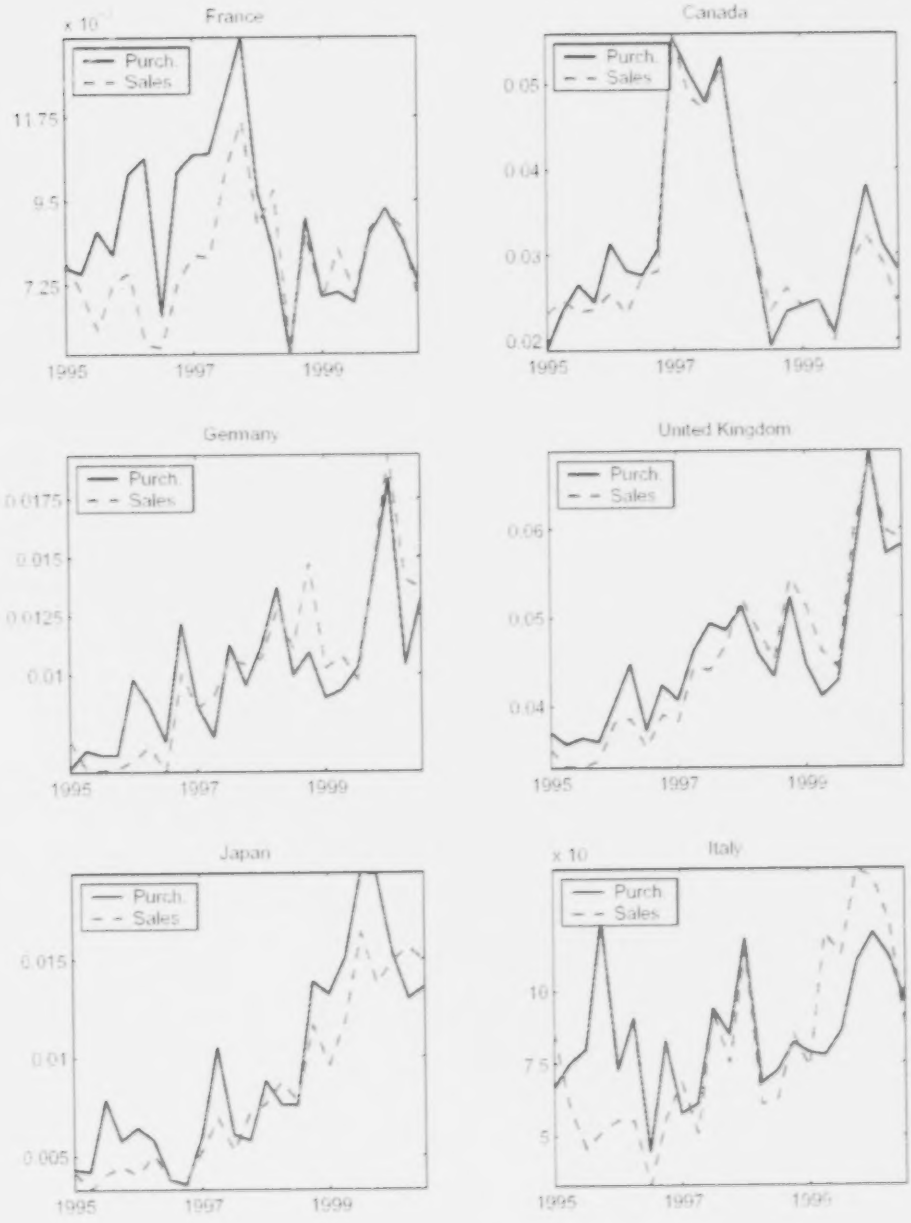


Figure 3: Autocorrelogram of Flows and Cross-Correlogram of Returns and Flows: France, Canada, and Germany. Notes: $\Delta\theta$ is net purchases of the local asset by U.S. investors; R is the current return on the local asset.

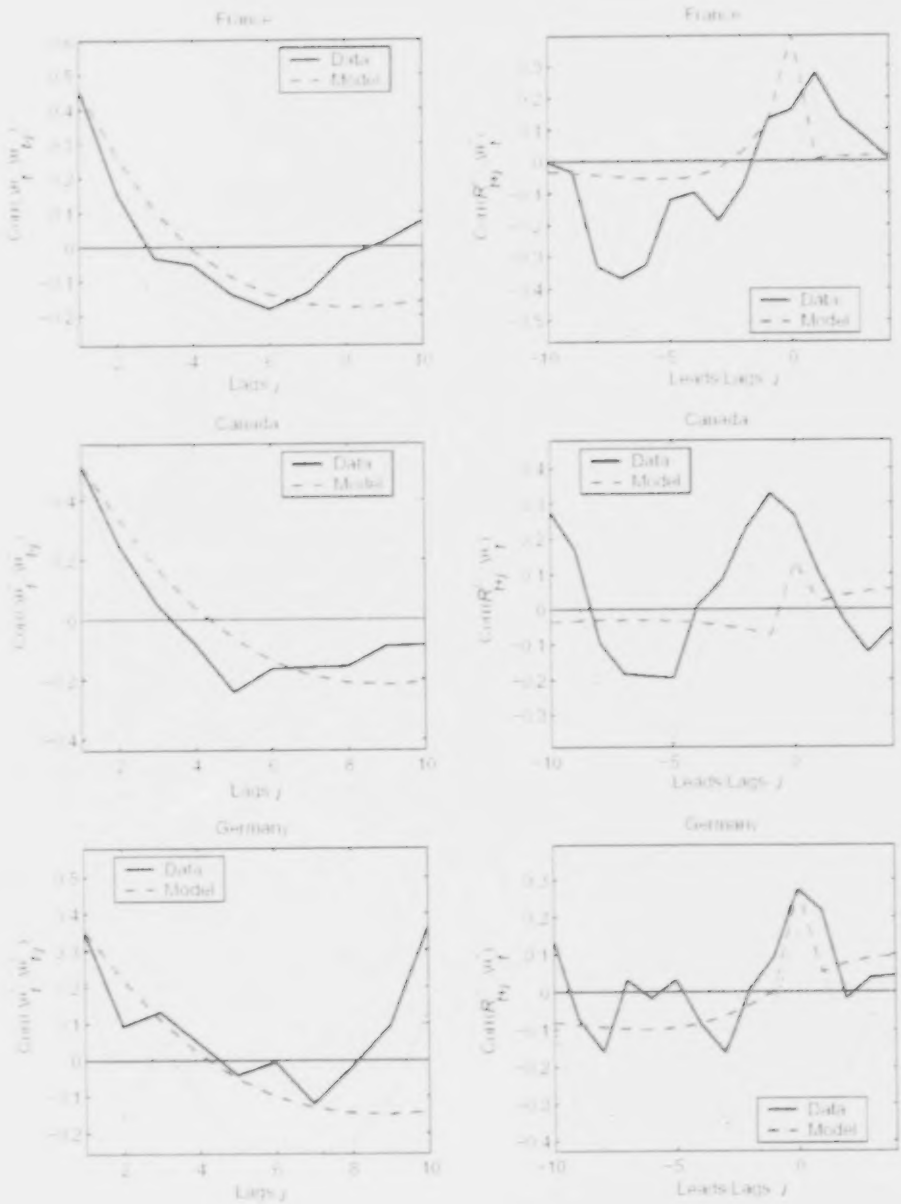


Figure 4: Autocorrelogram of Flows and Cross-Correlogram of Returns and Flows: U.K., Japan, and Italy. Notes: $\Delta\theta$ is net purchases of the local asset by U.S. investors; R is the current return on the local asset.

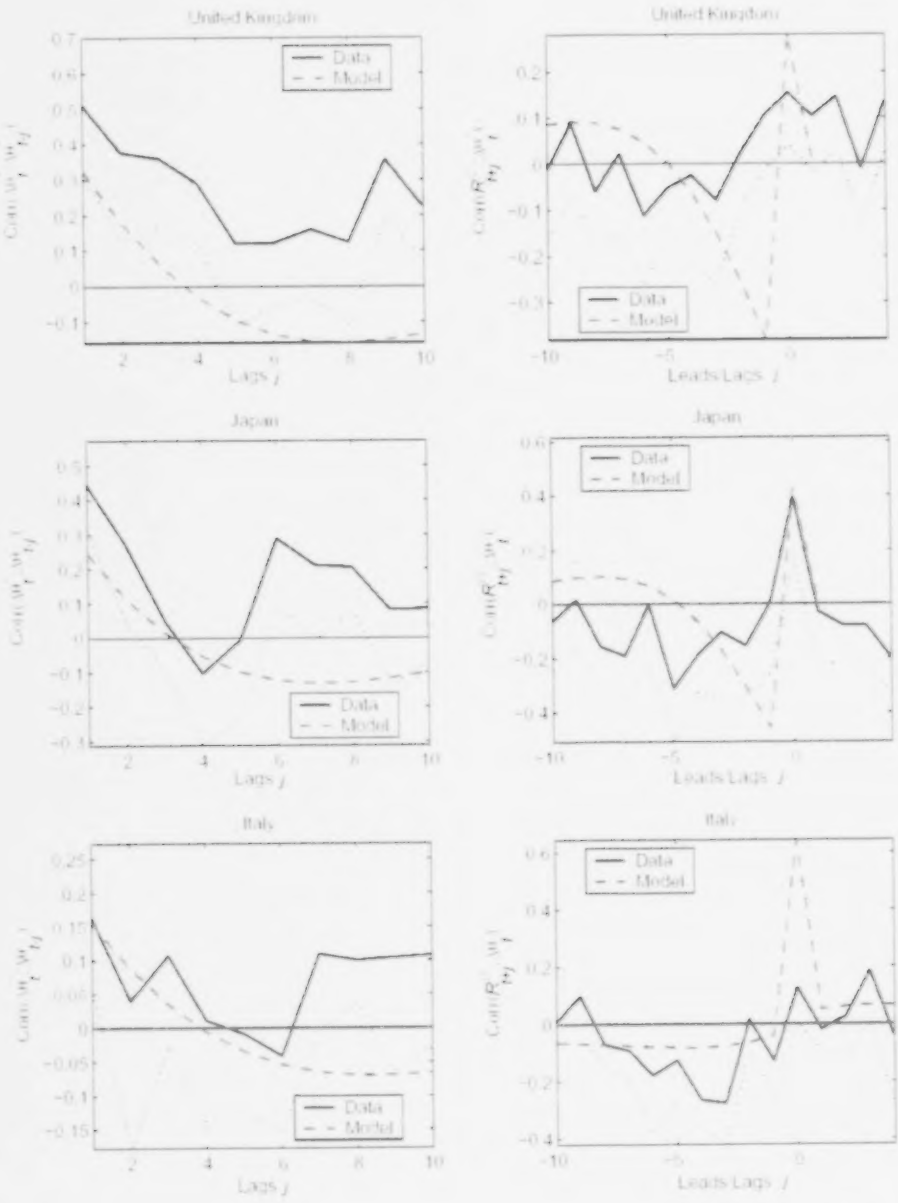


Figure 5: Impulse-Response Functions of the Asymmetric Information Model to a Persistent Business Cycle Shock. Notes: $\Delta\theta_t^i$ is net purchases of the local asset by unsophisticated investors; R_t is the current return on the local asset; $E_{t-1}^i(R_t)$ is the time t expectation by investors of type i of the time $t+1$ return on the local asset; $F_t^i - E_{t-1}^i(F_t^i)$ is the forecast error by investors of type i on the local business cycle factor.

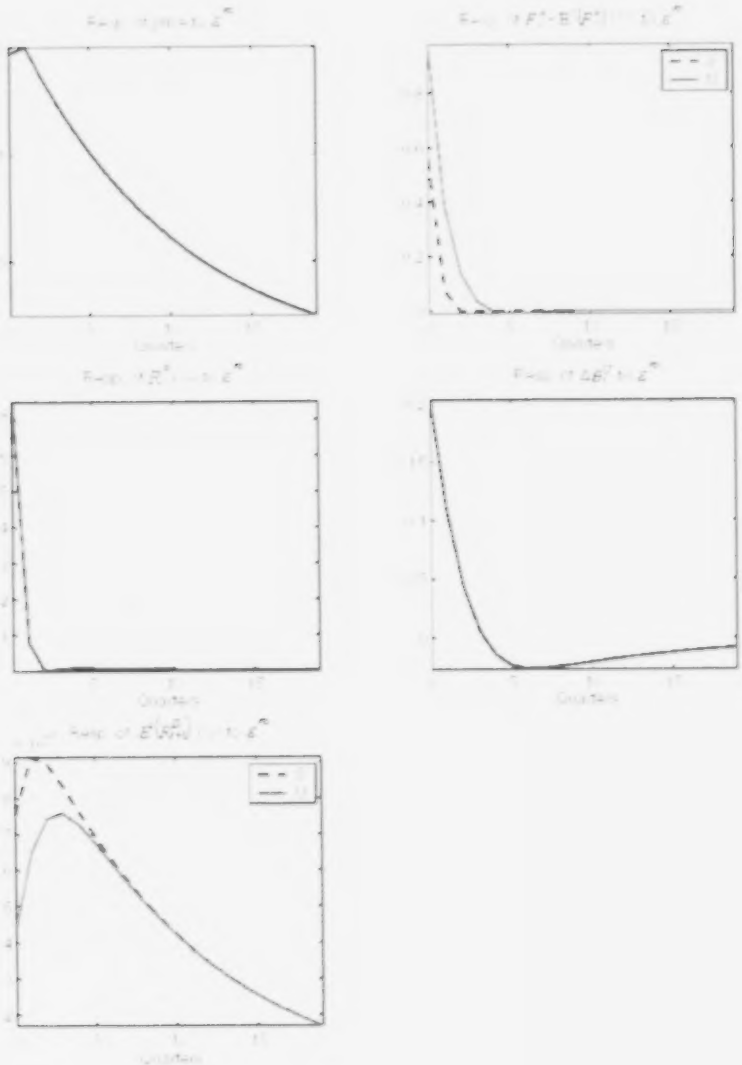


Figure 6: Impulse-Response Functions of the Asymmetric Information Model to a Transient Business Cycle Shock. Notes: $\Delta\theta_t^i$ is net purchases of the local asset by unsophisticated investors; R_t^L is the current return on the local asset; $E_t^i(R_{t+1}^L)$ is the time t expectation by investors of type i of the time $t+1$ return on the local asset; $F_t^L - E_t^i(F_t^L)$ is the forecast error by investors of type i on the local business cycle factor.

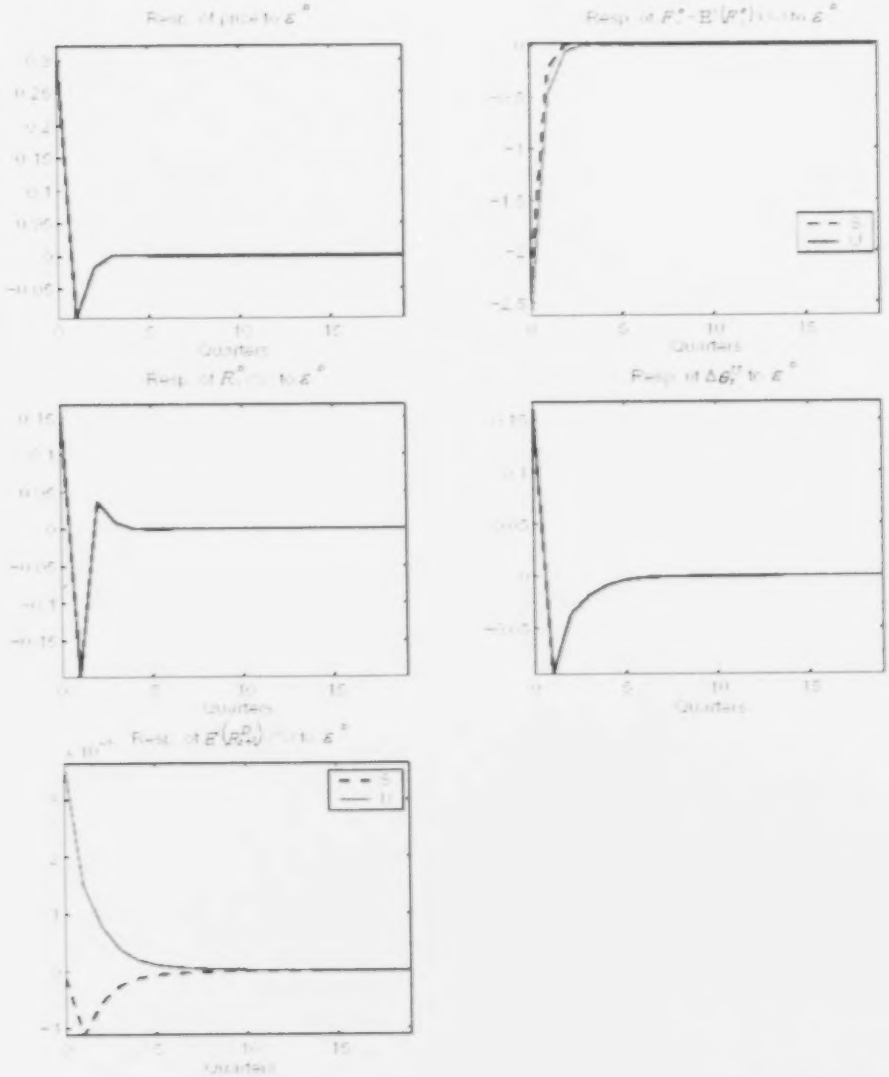


Figure 7: Impulse-Response Functions of the Asymmetric Information Model. Note: $\Delta\theta^{U}$ is the local business cycle factor.

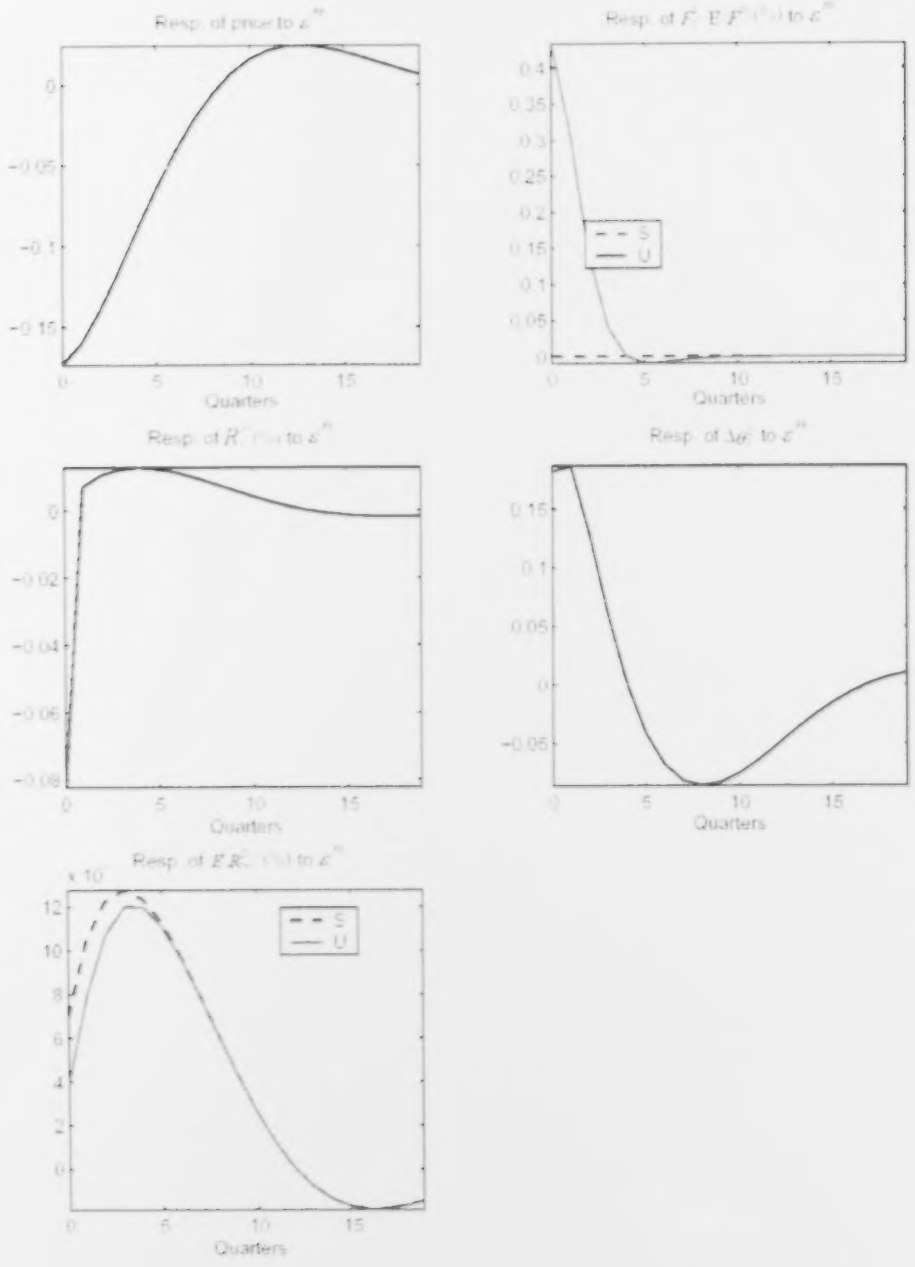
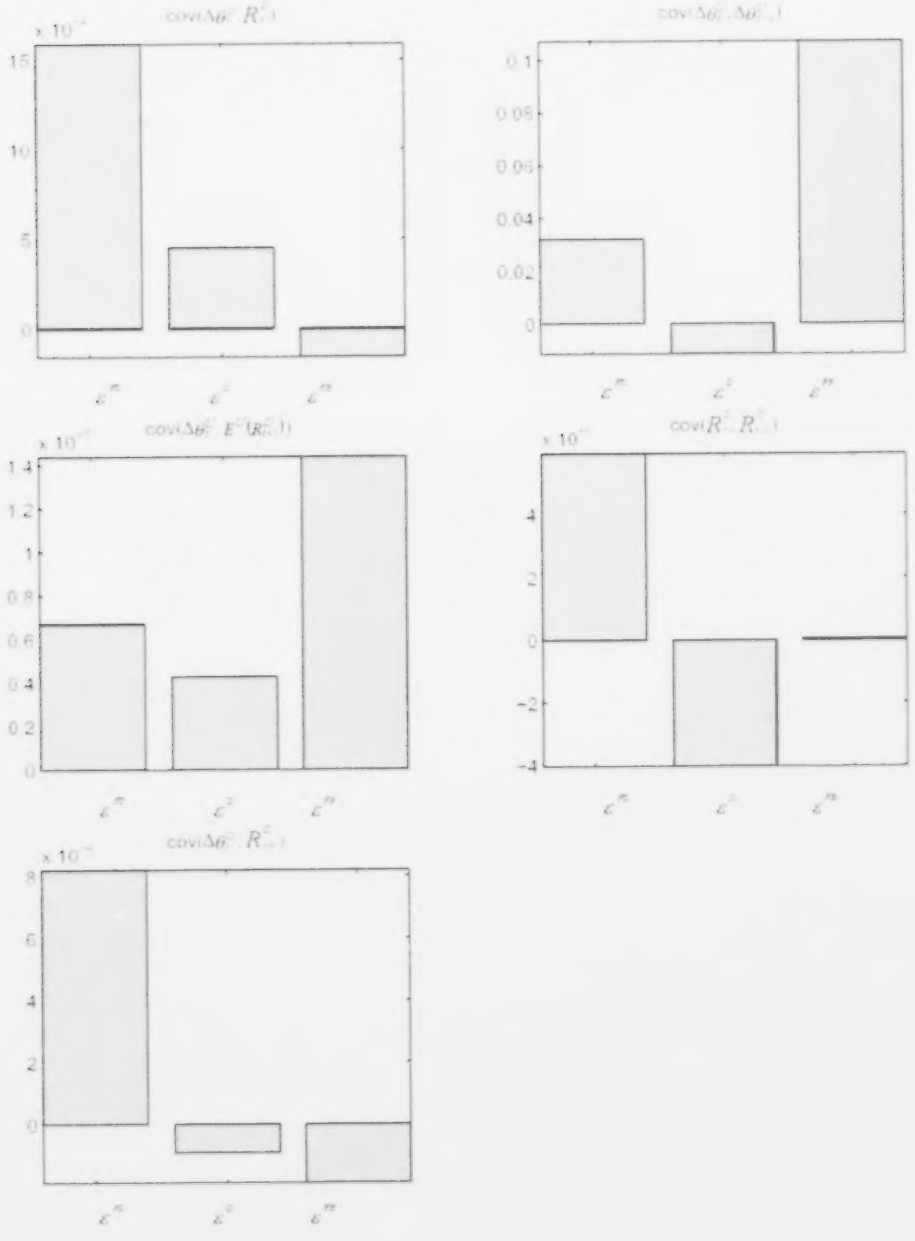


Figure 8: Variance Decomposition in the Asymmetric Information Model. Notes: $\Delta\theta_t^D$ is net purchases of the local asset by unsophisticated investors; R is the return on the local asset; E (R) is the time t expectation by unsophisticated investors of the time $t-1$ return on the local asset.



Appendix A: Proof of Theorem 1

In this appendix, we provide the complete proof of Theorem 1 in the main text. In the equilibrium that we analyze, the local equity asset price depends on factor realizations and beliefs of unsophisticated investors in these factors:

$$P_t = \bar{z} - \boldsymbol{\pi}_s' \hat{\mathbf{F}}_t^s + \boldsymbol{\pi}_u' \hat{\mathbf{F}}_t^u.$$

The vector $\mathbf{o}_t^u$, defined in the text, gives the vector of unsophisticated investors' observable variables; i.e., the local dividend and price and the world return. Unsophisticated investors do not see the return on sophisticated investors' private opportunities. Applying the Kalman filter to this problem for unsophisticated investors yields:

$$\hat{\mathbf{F}}_t = \boldsymbol{\varrho} \hat{\mathbf{F}}_{t-1} - \mathbf{K}^u \hat{\mathbf{u}}_t^u. \tag{A1}$$

with

$$E_t\left[\hat{\mathbf{u}}_t' \hat{\mathbf{u}}_t\right] = \tag{A2}$$
$$\mathbf{M}^{ou\prime} E_t\left[\left(\mathbf{F}_{t-1} - \hat{\mathbf{F}}_{t-1}^u\right)\left(\mathbf{F}_{t-1} - \hat{\mathbf{F}}_{t-1}^u\right)'\right] \mathbf{M}^{ou\prime\prime} + \mathbf{M}^{ou\varepsilon} E\left[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'\right] \mathbf{M}^{ou\varepsilon\prime}.$$

Using this information, we can now construct the unsophisticated investors' state vector, $\boldsymbol{\phi}_t^u = \hat{\mathbf{F}}_t^u$, and use (A1) and (A2) to derive its law of motion:

$$\boldsymbol{\phi}_{t+1}^u = \boldsymbol{\Phi}^u \boldsymbol{\phi}_t^u - \mathbf{M}^{o\varepsilon u} \boldsymbol{\varepsilon}_{t+1}^{ou}.$$

Repeating the same process for sophisticated investors' conditional forecasts, $\hat{\mathbf{F}}_t^s$, we have $\boldsymbol{\phi}_t^s = \left(\hat{\mathbf{F}}_t^{s\prime}, \hat{\mathbf{F}}_t^{u\prime}\right)'$:

$$\boldsymbol{\phi}_{t+1}^s = \boldsymbol{\Phi}^s \boldsymbol{\phi}_t^s - \mathbf{M}^{o\varepsilon s} \boldsymbol{\varepsilon}_{t+1}^{ou}.$$

Consider the decision problem of both investors. Write returns as

$$\mathbf{R}_{t+1}^i = \bar{\mathbf{R}}^i - \mathbf{M}^{Ro\phi i} \boldsymbol{\phi}_t^i - \mathbf{M}^{Ro\varepsilon i} \boldsymbol{\varepsilon}_{t+1}^{ou},$$

for each investor. Guess that investors' i value function is of the form

$$V^i\left(w_t^i; \boldsymbol{\phi}_t^i\right) = -\exp\left[-\kappa^i - \tilde{\gamma} w_t^i - \mathbf{u}^i \boldsymbol{\phi}_t^i - \frac{1}{2} \boldsymbol{\phi}_t^{i\prime} \mathbf{U}^i \boldsymbol{\phi}_t^i\right].$$

Define $\mathbf{\Omega}^i = \left(\mathbf{M}^{\varepsilon\varepsilon i\prime}\mathbf{U}_i\mathbf{M}^{\varepsilon\varepsilon i} + \left(\mathbf{\Sigma}^i_{\varepsilon\varepsilon}\right)^{-1}\right)^{-1}$, where $\mathbf{\Sigma}^i_{\varepsilon\varepsilon} = E\left[\boldsymbol{\varepsilon}^{\varepsilon i}_t \boldsymbol{\varepsilon}^{\varepsilon i\prime}_t\right]$. We have that (superscript i dropped for simplicity):

$$\begin{aligned} E_t V\left(w_{t+1}, \boldsymbol{\phi}_{t+1}\right) &= -\frac{\left(\det \mathbf{\Sigma}_{\varepsilon\varepsilon}\right)^{-\frac{1}{2}}}{\left(\det \mathbf{\Omega}\right)^{-\frac{1}{2}}} \cdot \\ &\quad \exp\left(-\kappa - \tilde{\gamma}\left(R_f\left(w_t - c_t\right) + \boldsymbol{\theta}'_t(\bar{\mathbf{R}} + \mathbf{M}^{R\varepsilon}\boldsymbol{\phi}_t)\right)\right) \\ &\quad \exp\left(-\frac{1}{2}\boldsymbol{\phi}'_t\mathbf{\Phi}'\mathbf{U}\mathbf{\Phi}\boldsymbol{\phi}_t - \mathbf{u}'\mathbf{\Phi}\boldsymbol{\phi}_t\right) \\ &\quad \exp\left(\begin{array}{l} \frac{1}{2}\left(\tilde{\gamma}\boldsymbol{\theta}'_t\mathbf{M}^{R\varepsilon\varepsilon} + \left(\boldsymbol{\phi}'_t\mathbf{\Phi}'\mathbf{U} + \mathbf{u}'\right)\mathbf{M}^{\varepsilon\varepsilon}\right) \cdot \\ \mathbf{\Omega}\left(\tilde{\gamma}\mathbf{M}^{R\varepsilon\varepsilon\prime}\boldsymbol{\theta}_t + \mathbf{M}^{\varepsilon\varepsilon\prime}\left(\mathbf{U}\mathbf{\Phi}\boldsymbol{\phi}_t + \mathbf{u}\right)\right) \end{array}\right). \end{aligned}$$

Solving for the optimal portfolio, we obtain:

$$\begin{aligned} \boldsymbol{\theta}_t &= \tilde{\gamma}^{-1}\left(\mathbf{M}^{\theta\theta}\right)^{-1}\left(\overline{\mathbf{M}}^{\theta} + \mathbf{M}^{\theta\varepsilon}\boldsymbol{\phi}_t\right) \\ &= \bar{\boldsymbol{\theta}} - \mathbf{\Theta}\boldsymbol{\phi}_t, \end{aligned}$$

where the matrices are given by

$$\begin{aligned} \overline{\mathbf{M}}^{\theta\prime} &= \bar{\mathbf{R}}' - \mathbf{u}'\mathbf{M}^{\varepsilon\varepsilon}\mathbf{\Omega}\mathbf{M}^{R\varepsilon\varepsilon\prime}, \\ \mathbf{M}^{\varepsilon\theta\prime} &= \mathbf{M}^{R\varepsilon\prime} - \mathbf{\Phi}'\mathbf{U}\mathbf{M}^{\varepsilon\varepsilon}\mathbf{\Omega}\mathbf{M}^{R\varepsilon\varepsilon\prime}, \end{aligned}$$

and

$$\mathbf{M}^{\theta\theta} = \mathbf{M}^{R\varepsilon\varepsilon}\mathbf{\Omega}\mathbf{M}^{R\varepsilon\varepsilon\prime}.$$

The first term (i.e., $\tilde{\gamma}^{-1}\left(\mathbf{M}^{\theta\theta}\right)^{-1}\mathbf{M}^{R\varepsilon}$) in matrix $\mathbf{\Theta}$ gives the myopic demand of the investor, whereas the second term (i.e., $-\tilde{\gamma}^{-1}\left(\mathbf{M}^{\theta\theta}\right)^{-1}\mathbf{M}^{R\varepsilon\varepsilon}\mathbf{\Omega}\mathbf{M}^{\varepsilon\varepsilon\prime}\mathbf{U}\mathbf{\Phi}$) gives the hedging demand of the investor.

From the value function $V\left(w'_t; \boldsymbol{\phi}'_t\right)$, we see that risk-averse investors care not only about fluctuations in wealth, but also about changes in beliefs, captured by the state vector, $\boldsymbol{\phi}'_t$. The quadratic term reflects investors' taste for "unusual" investment opportunities. Intuition for this effect can be obtained by considering the case of one state variable. $\mathbf{U}_i$ is then a positive number and continuation utility is higher the further $\boldsymbol{\phi}'_t$ is from its mean of zero. Since $\boldsymbol{\phi}'_t$ is payoff-relevant, it drives expected returns at some time in the future. An unusual value signals that above-average expected returns will be available, by either going long or short.

The foregoing enables us to describe in detail the coefficients of the optimal portfolio policy, $\theta_t^i = \bar{\theta}^i - \Theta^i \phi_t^i$. We have

$$\begin{aligned}
\theta_t^i &= \tilde{\gamma}^{-1} \tilde{\Sigma}_{R^i R^i}^{-1} E^i \left(\mathrm{R}_{t+1}^i | \phi_t^i \right) \\
&\quad - \tilde{\gamma}^{-1} \tilde{\Sigma}_{R^i R^i}^{-1} Cov^i \left(\left(\mathrm{u}_i' + E^i \left[\phi_{t+1}^i | \phi_t^i \right]' \mathrm{U}_i \right) \phi_{t+1}^i, \mathrm{R}_{t+1}^i | \phi_t^i \right) \quad \text{(A3)} \\
&= \tilde{\gamma}^{-1} \tilde{\Sigma}_{R^i R^i}^{-1} \left(E^i \left(\mathrm{R}_{t+1}^i | \phi_t^i \right) - \left(\bar{\mathrm{h}}^i - H^i \phi_t^i \right) \right). \quad \text{(A4)}
\end{aligned}$$

where the matrix $\tilde{\Sigma}_{R^i R^i}$ is a transformation of the conditional covariance matrix of returns.[42] We use this decomposition in the main text.

Solving for the optimal consumption level and the value function, we see that, for a given price function, optimality requires the following constraints to be met: $\tilde{\gamma} = \gamma \frac{R_f - 1}{R_f}$, and

$$\begin{aligned}
\kappa &= -\log\left(\frac{R_f}{R_f - 1}\right) \\
&\quad - \frac{1}{R_f - 1} \frac{1}{2} \left(\log\left(\frac{\det \Omega}{\det \Sigma_{\infty\infty}} \right) - \mathrm{u}' \mathrm{M}^{\infty\beta} \Omega \mathrm{M}^{\infty\beta\prime} \mathrm{u} - \overline{\mathrm{M}}^{\beta\prime} \left(\mathrm{M}^{\beta\beta} \right)^{-1} \overline{\mathrm{M}}^{\beta} \right) \\
\mathrm{u} &= \frac{1}{R_f} \left(\overline{\mathrm{M}}^{\infty} - \mathrm{M}^{\beta\infty\prime} \left(\mathrm{M}^{\beta\beta} \right)^{-1} \overline{\mathrm{M}}^{\beta} \right) \\
\mathrm{U} &= \frac{1}{R_f} \left(\mathrm{M}^{\infty\infty} - \mathrm{M}^{\beta\infty\prime} \left(\mathrm{M}^{\beta\beta} \right)^{-1} \mathrm{M}^{\beta\infty} \right),
\end{aligned}$$

with $\overline{\mathrm{M}}^{\beta\prime} = \mathrm{u}' \left(\mathrm{I} - \mathrm{M}^{\infty} \Omega \mathrm{M}^{\infty\beta} \mathrm{U} \right) \Phi$ and $\mathrm{M}^{\infty\infty} = \Phi' \mathrm{U} \left(\mathrm{I} - \mathrm{M}^{\infty} \Omega \mathrm{M}^{\infty\prime} \mathrm{U} \right) \Phi$.

Finally, to solve for an equilibrium, let $\Theta_{f^\infty}^{Da}$ be the part of the first row of Θ^∞ that is associated with $\tilde{\mathrm{F}}_t^\infty$, and let Θ^{Da} be the first row of Θ^∞. In equilibrium, we require that

$$\begin{aligned}
\Delta \bar{\theta}^{Da} + (1 - \Delta) \bar{\theta}^{Da} &= 1 \\
\Delta \Theta^{Da} + (1 - \Delta) \Theta_f^{Da} &= 0.
\end{aligned}$$

[42] Formally, we define matrix $\mathrm{M}^{R^i \infty}$ such that $\mathrm{R}_t^i = \mathrm{M}^{R^i \infty} \phi_t^i$. Then,

$$\tilde{\Sigma}_{R^i R^i} = \mathrm{M}^{R^i \infty} \left(Var\left(\phi_{t+1}^i | \phi_t^i \right)^{-1} - \mathrm{U}_i \right)^{-1} \mathrm{M}^{R^i \infty\prime}$$

Appendix B: Detrending

In the data, dividends and flows exhibit trends, whereas our quantitative exercise explores a detrended economy. In this appendix, we outline a consistent approach to detrending dividends and flows. To fix ideas, consider the following stylized view of the stock market. There are S firms, each with a single share, paying the same (per-share) dividend, $D_t^\star$, and having the same (per-share) price, $P_t^\star$. Dividends grow at an exponential rate, η. The parameter η thus captures trend firm productivity growth, which benefits owners through dividends.

An observed aggregate price index records the change in the value of the average firm, $P_t^\star / P_{t-1}^\star$. This change in valuation has two components: capital gains that arise from fluctuations in the firm's *stationary* price, P_t / P_{t-1}, and the growth in prices built-in from productivity growth:

$$\frac{P_t^\star}{P_{t-1}^\star} = e^{\eta} \frac{P_t}{P_{t-1}}.$$

Observed dividend yield, δ_t, is simply the ratio $D_t^\star / P_t^\star = D_t / P_t$. A natural way to remove the trend from dividends is to exponentially detrend the measure $\delta_t P_t^\star$. Observed holdings of the domestic equity index by investor i are $P_t^\star \theta_t^{D_i \star}$. Observed market capitalization at the end of period t is the combined value of all plants, $M_t^\star = P_t^\star \bar{S}$. Normalizing holdings by beginning-of-period market capitalization is thus a natural way to remove the exponential trend in holdings. The normalized holdings are:

$$\theta_t^{D_i} = \frac{\theta_t^{D_i \star} P_t^\star}{\bar{S} P_t^\star}.$$

There is an explicit connection between dividends and equilibrium holdings before and after detrending. We can summarize an economy driven by trending exogenous variables using a tuple, $\mathcal{E} = (R_f^\star, \bar{S}, (D_t^\star, R_t^{W\star}, R_t^{B\star})_{t=0}^{\infty})$. Suppose that $D_t^\star = e^{\eta t} D_t$ and that $(P_t^\star, \theta_t^{D\star}, \theta_t^{W\star}, \theta_t^{B\star}, c_t^\star)$ is an equilibrium of $\mathcal{E}$.[33] It can be verified that the tuple

$$\left(P_t, \theta_t^D, e^{-\eta t} \theta_t^{W\star} / \bar{S}, e^{-\eta t} \theta_t^{B\star}, e^{-\eta t} c_t^\star\right),$$

[33] We suppress the indexes for the different types of agent. Naturally, holdings of the same asset by different types of agent are detrended in the same way.

is an equilibrium of the detrended economy

$$\mathcal{E}_\eta = \left(R_f^* e^{-\eta}, 1, \left(D_t, e^{-\eta} R_t^{W*}, e^{-\eta} R_t^{R*}\right)_{t=0}^{\infty}\right).$$

In our quantitative exercise, we consider a detrended economy. We determine a stationary dividend process, D_t, as the residual in a regression of average firm dividends on a time trend.

$$\log\left(\delta_t P_t^*\right) = E\left[\log D_t\right] - \eta t - \left(\log D_t - E\left[\log D_t\right]\right). \tag{B1}$$

We then match the equilibrium flows to observed flows normalized by market capitalization. In the light of the above result, this ensures consistent detrending of dividends and flows.

We also need to select an interest rate, R_f, and a return process, R_t^W, for the detrended economy. We use the observed average interest rate and U.S. stock return. In terms of the above notation, we are thus analyzing the economy $\mathcal{E}_0$. Given our data, this is preferable to considering the economy $\mathcal{E}_{\hat{\eta}}$, where $\hat{\eta}$ is the growth-rate estimate from (B1): in a small sample such as ours, $\hat{\eta}$ is driven by medium-term developments and does not reflect the long-run average growth rate. In particular, in our sample, $\hat{\eta}$ exceeds the average real riskless interest rate. We are thus not likely to learn much by considering equilibrium flows from $\mathcal{E}_{\hat{\eta}}$. At the same time, the result of the previous paragraph shows that the only role of the trend growth rate, η, is to shift *all* returns, which suggests that the behaviour of the correlations we are interested in will be similar across all economies $\mathcal{E}_\eta$ where η is reasonably small.

Appendix C: The Dividend Process

In this appendix, we describe how we estimate the dividend process. We derive conditions under which a general ARMA(2,2) process permits a representation of the type we assume for our dividend process:

$$\begin{aligned} F_t^D &= a_1 F_{t-1}^D + a_2 F_{t-2}^D + \varepsilon_t^{FD}, \\ D_t &= \bar{D} + F_{t-1}^D + \varepsilon_t^D, \end{aligned} \tag{C1}$$

where ε_t^{FD} and ε_t^D are serially uncorrelated and independent random variables with zero mean and $E\left[\left(\varepsilon_t^{FD}\right)^2\right] = \sigma_{\varepsilon FD}^2$, $E\left[\left(\varepsilon_t^D\right)^2\right] = \sigma_{\varepsilon D}^2$. To prove our result, we need to compare the correlogram of dividends under the two representations. Consider first the representation, (C1). The correlogram of the persistent component, F_t^D, is summarized by

$$\begin{aligned} Var\left(F_t^D\right) &= \left(1 - a_1^2 - a_2^2 - \frac{2a_2a_1^2}{1-a_2}\right)^{-1} \sigma_{\varepsilon FD}^2, \\ Cov\left(F_t^D, F_{t-1}^D\right) &= \frac{a_1}{1-a_2} Var\left(F_t^D\right), \\ Cov\left(F_t^D, F_{t-2}^D\right) &= a_1 Cov\left(F_t^D, F_{t-1}^D\right) + a_2 Var\left(F_t^D\right) \\ &= \left(\frac{a_1^2}{1-a_2} + a_2\right) Var\left(F_t^D\right), \\ Cov\left(F_t^D, F_{t-s}^D\right) &= a_1 Cov\left(F_t^D, F_{t-s+1}^D\right) + a_2 Cov\left(F_t^D, F_{t-s+2}^D\right) : \; s \geq 3. \end{aligned}$$

The correlogram of the dividend process is thus given by

$$\begin{aligned} Var\left(D_t - \bar{D}\right) &= Var\left(F_t^D\right) + \sigma_{\varepsilon D}^2, \\ Cov\left(D_t - \bar{D}, D_{t-1} - \bar{D}\right) &= Cov\left(F_t^D, F_{t-1}^D\right) \\ &= \frac{a_1}{1-a_2}\left[Var\left(D_t - \bar{D}\right) - \sigma_{\varepsilon D}^2\right], \\ Cov\left(D_t - \bar{D}, D_{t-2} - \bar{D}\right) &= Cov\left(F_t^D, F_{t-2}^D\right) \\ &= a_1 Cov\left(D_t - \bar{D}, D_{t-1} - \bar{D}\right) \\ &\quad + a_2\left[Var\left(D_t - \bar{D}\right) - \sigma_{\varepsilon D}^2\right], \end{aligned}$$

as well as, for every $s \geq 3$,

$$\begin{aligned} Cov\left(D_t - \bar{D}, D_{t-s} - \bar{D}\right) &= Cov\left(F_t^D, F_{t-s}^D\right) \\ &= a_1 Cov\left(D_t - \bar{D}, D_{t-s+1} - \bar{D}\right) \\ &\quad + a_2 Cov\left(D_t - \bar{D}, D_{t-s+2} - \bar{D}\right). \end{aligned}$$

Next, consider a general ARMA(2,2) process

$$D_t - \bar{D} = a_1 \left(D_{t-1} - \bar{D} \right) + a_2 \left(D_{t-2} - \bar{D} \right) + u_t - \lambda_1 u_{t-1} + \lambda_2 u_{t-2},$$

where u_t is serially uncorrelated with mean zero and variance σ_u^2. Squaring both sides and taking expectations, we have

$$\begin{aligned} Var\left(D_t - \bar{D} \right) &= a_1^2 Var\left(D_{t-1} - \bar{D} \right) + a_2^2 Var\left(D_{t-2} - \bar{D} \right) + \sigma_u^2 \left(1 + \lambda_1^2 + \lambda_2^2 \right) \\ &\quad - 2a_1 \lambda_1 Cov\left(D_{t-1} - \bar{D}, u_{t-1} \right) + 2a_1 \lambda_2 Cov\left(D_{t-1} - \bar{D}, u_{t-2} \right) \\ &\quad - 2a_2 \lambda_2 Cov\left(D_{t-2} - \bar{D}, u_{t-2} \right) \\ &\quad + 2a_1 a_2 Cov\left(D_t - \bar{D}, D_{t-1} - \bar{D} \right). \end{aligned}$$

Multiplying both sides by $\left(D_{t-1} - \bar{D} \right)$ and taking expectations, we have

$$\begin{aligned} Cov\left(D_t - \bar{D}, D_{t-1} - \bar{D} \right) &= Cov\left(\begin{array}{c} a_1 \left(D_{t-1} - \bar{D} \right) + a_2 \left(D_{t-2} - \bar{D} \right) \\ - \lambda_1 u_{t-1} - \lambda_2 u_{t-2}, D_{t-1} - \bar{D} \end{array} \right) \\ &= \frac{a_1}{1 - a_2} Var\left(D_t - \bar{D} \right) \\ &\quad + \frac{\lambda_1 - \lambda_2 \lambda_1 - \lambda_2 a_1}{1 - a_2} \sigma_u^2, \end{aligned} \tag{C2}$$

Finally, multiplying by $\left(D_{t-2} - \bar{D} \right)$ and taking expectations, we obtain

$$\begin{aligned} Cov\left(D_t - \bar{D}, D_{t-2} - \bar{D} \right) &= a_1 Cov\left(D_t - \bar{D}, D_{t-1} - \bar{D} \right) \\ &\quad + a_2 Var\left(D_t - \bar{D} \right) + \lambda_2 \sigma_u^2. \end{aligned} \tag{C3}$$

The variance can be solved in terms of parameters only:

$$\begin{aligned} Var\left(D_t - \bar{D} \right) &= \sigma_u^2 \left(1 - a_1^2 - a_2^2 - \frac{2a_1^2 a_2}{1 - a_2} \right)^{-1} \\ &\quad \times \left(\begin{array}{c} 1 + \lambda_1^2 + \lambda_2^2 + 2a_1 \lambda_1 + 2a_1^2 \lambda_2 - 2a_1 \lambda_2 \lambda_1 + 2a_2 \lambda_2 \\ - 2a_1 a_2 \frac{\lambda_1 + \lambda_2 \lambda_1 + \lambda_2 a_1}{1 - a_2} \end{array} \right). \end{aligned}$$

The first and second covariances are then given by (C2) and (C3) and all further covariances (for $s \geq 3$) follow the recursion

$$\begin{aligned} Cov\left(D_t - \bar{D}, D_{t-s} - \bar{D} \right) &= a_1 Cov\left(D_t - \bar{D}, D_{t-s+1} - \bar{D} \right) \\ &\quad + a_2 Cov\left(D_t - \bar{D}, D_{t-s+2} - \bar{D} \right). \end{aligned}$$

It is clear that, if a given ARMA(2,2) process is to have the representation (C1), the autoregressive coefficients must be the same in both representations. Moreover, since the recursions for all covariances beyond lag 2 are identical, a representation of the type (C1) exists if there exist $\sigma^2_{\varepsilon FD}, \sigma^2_{\varepsilon D} > 0$ such that the variance and the first two covariances are matched, which requires that:

$$\begin{aligned}
&\sigma^2_u \begin{pmatrix} 1 - \lambda_1^2 - \lambda_2^2 + 2a_1\lambda_1 + 2a_1^2\lambda_2 \\ -2a_1\lambda_2\lambda_1 + 2a_2\lambda_2 + 2a_1a_2\frac{\lambda_1+\lambda_2\lambda_1-\lambda_2a_1}{1-a_2} \end{pmatrix} \\
= \;& \sigma^2_{\varepsilon FD} + \sigma^2_{\varepsilon D}\left(1 - a_1^2 - a_2^2 - \frac{2a_2a_1^2}{1-a_2}\right).
\end{aligned}$$

$$(\lambda_1 + \lambda_2\lambda_1 + \lambda_2 a_1)\,\sigma^2_u = -a_1\sigma^2_{\varepsilon D}.$$

$$\lambda_2\sigma^2_u = -a_2\sigma^2_{\varepsilon D}.$$

The first and last equations can be used to calculate the implied values of $\sigma^2_{\varepsilon D}$ and $\sigma^2_{\varepsilon FD}$, and obtain two inequality constraints on the ARMA(2,2) parameters:

$$\begin{aligned}
\sigma^2_{\varepsilon D} &= -\frac{\lambda_2}{a_2}\sigma^2_u > 0, \\
\sigma^2_{\varepsilon FD} &= \sigma^2_u\Big[1 - \lambda_1^2 - \lambda_2^2 + 2a_1\lambda_1 + 2a_1^2\lambda_2 - 2a_1\lambda_2\lambda_1 - 2a_2\lambda_2 \\
&\qquad -2a_1a_2\tfrac{\lambda_1+\lambda_2\lambda_1+\lambda_2a_1}{1-a_2} \\
&\qquad +\tfrac{\lambda_2}{a_2}\left(1 - a_1^2 - a_2^2 - \tfrac{2a_2a_1^2}{1-a_2}\right)\Big] > 0.
\end{aligned} \tag{C4}$$

The second equation implies the additional constraint

$$0 = a_2\lambda_1(1+\lambda_2) - a_1\lambda_2(1-a_2). \tag{C5}$$

In a first estimation step, we impose (C5), but do not impose the inequality constraint. The inequalities are not binding in all countries except for Japan and the United Kingdom. For these countries, we impose $\sigma^2_{\varepsilon D} = 0.001$, and re-estimate the restricted ARMA(2,2) process. Setting the variance of transient shocks to dividends equal to zero implies that there are no trades based on private information because the equilibrium is fully revealing.

Table C1 reports the estimates for the restricted ARMA(2,2) process. These estimates are then used to produce Table 2 in the main text according to the formulas in (C4). The estimated ARMA(2,2) produces statistically

significant estimates of the autoregressive parameters a_1 and a_2 for most of the countries (except for Japan's a_2), and statistically significant estimates of the moving-average parameters λ_1 and λ_2 as well (except for France and Japan). Estimates of σ_ε^2 are also significant in all cases except for Canada. The constraint (C5) is not rejected in four out of nine countries at the usual 5 per cent significance level.

Table C1: Estimates of ARMA(2,2) Process

	a_1	a_2	λ_1	λ_2	σ_u^2	χ^2 *p*-value
Canada	1.859	-0.896	-1.051	0.365	0.013	4.499
	6.59	-3.94	-4.23	2.91	0.78	0.033
France	1.369	-0.420	-0.092	0.020	0.014	5.327
	33.39	-20.60	-0.64	0.62	5.42	0.020
Germany	1.734	-0.773	-0.803	0.253	0.101	2.608
	19.47	-8.99	-4.92	3.80	6.18	0.106
Italy	1.685	-0.708	-0.398	0.108	0.001	30.41
	51.41	-32.03	-4.94	4.47	8.28	0.000
Japan	1.212	-0.275	-0.002	0.0004	0.786	2.768
	4.884	-0.295	-0.815	0.272	3.141	0.096
U.K.	1.223	-0.294	-0.003	0.0005	0.575	2.089
	16.408	-5.464	-9.349	6.125	8.212	0.148
U.S.	1.679	-0.747	-0.754	0.237	2.100	3.846
	6.60	-3.18	-2.02	1.55	9.48	0.049

Note: For each country, the second row gives *t*-statistics for the corresponding estimates. $\chi^2(1)$ and *p*-values are given for the non-linear constraint (C5).

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